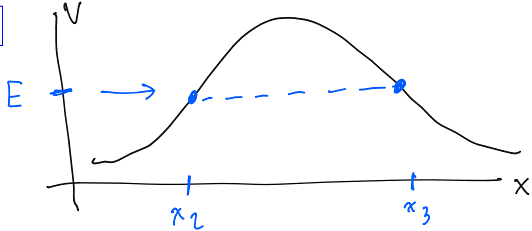
PHYS 4410
Quantum Mechanics 2
Spring 2023

Lecture 36
Metastable states

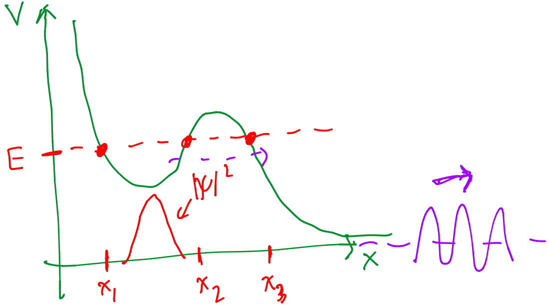
April 24



$$P_{tunnel} = e^{-2\Gamma}$$

$$\Gamma \approx \int_{x_2}^{x_3} dx \, \frac{\sqrt{2m(V(x)-E)}}{\hbar}$$

Today: metastability



What happens as $t \to \infty$?

- state btwn x_1 & x_2 cannot be true bound state
- exact eigenstates unbound
- "almost" bound state ...

metastable.

$\boxed{2}$ Lifetime of metastable state?

- each time packet hits x_2, prob $P_{tunnel} = e^{-2\Gamma}$ to escape

At time t, $P_{12} = \int_{x_1}^{x_2} dx\, |\psi|^2$:

$$\frac{dP_{12}}{dt} = -P_{tunnel} \cdot P_{12} \cdot \frac{1}{T}$$

P_{tunnel} ↑ prob. to tunnel per collision

P_{12} ↑ haven't left

$\frac{1}{T}$ ↑ time btwn collisions: period of osc. (lec 33)

For time scales $t \gg T$

$P_{12} \approx e^{-t/\tau}$

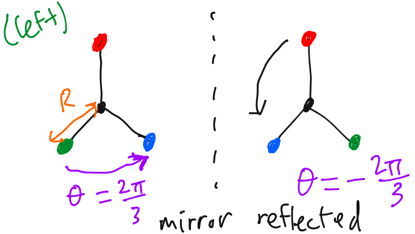
where <u>lifetime</u>, τ:

$$\tau = \frac{T}{P_{tunnel}} = T e^{2\Gamma}$$

<u>Caution</u>: $e^{2\Gamma}$ makes it hard to quantitatively <u>predict</u>

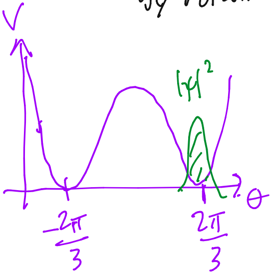
Often: $T \sim \frac{|x_2 - x_1|}{v_{typ}} \sim |x_2 - x_1| \Big/ \sqrt{\frac{2\Delta E}{m}}$ ← kinetic energy

3 Example: chiral molecules (in 2d)



mirror reflected

Configs not related by rotation



Estimate: $-\frac{\hbar^2}{2m}\frac{d^2}{dx^2}$

$$H = -\frac{\hbar^2}{2I}\frac{\partial^2}{\partial\theta^2} + K\left(|\theta| - \frac{2\pi}{3}\right)^2.$$

$I \sim MR^2 \sim 10^{-45}$ kg·m^2

$M \sim 10^{-25}$ kg

$R \sim 10^{-10}$ m

units of energy:

$K \sim 5$ eV $\sim 10^{-18}$ J

Toy H is $\sim$ oscillator:

$m \to I$

$m\omega^2 \to K \longrightarrow \omega = \sqrt{\frac{K}{I}}$

$\boxed{4}$ Fluctuations in θ near $\theta = 2\pi/3$?

$$\Delta\theta_{typ} \sim \text{"}\sqrt{\frac{\hbar}{m\omega}}\text{"} \sim \sqrt{\frac{\hbar}{I\sqrt{\frac{K}{I}}}} = \frac{\sqrt{\hbar}}{(KI)^{1/4}} \lesssim \frac{1}{20}.$$

$(\hbar \sim 10^{-34}\ J\cdot s)$

Tunneling time?

$$\tau \sim T e^{2\Gamma}$$

$$\Gamma = \int_{-2\pi/3}^{2\pi/3} \frac{d\theta}{\hbar} \sqrt{2I\cdot K(|\theta| - 2\pi/3)^2}$$

$$= \frac{2\sqrt{2IK}}{\hbar} \int_0^{2\pi/3} d\theta \left(\frac{2\pi}{3} - \theta\right) \sim \frac{10}{\Delta\theta_{typ}^2}$$

$$\sim 4\times 10^3$$

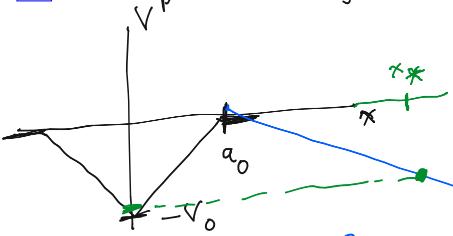
for harmonic oscillator:

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{I}{K}}$$

$$\sim 10^{-13}\ s$$

$$\tau \sim 10^{-13} e^{4000} \gg 10^{17}\ s$$ (universe age)

$\boxed{5}$ Example: ionizing atoms.



$\mathcal{E}$ small

$x_* \gg a_0$

$x_* \approx \dfrac{V_0}{q\mathcal{E}}$

Turn on electric field? $V \to V - q\mathcal{E}(x - a_0)$

Estimate Γ?

$$\Gamma \approx \int_{0\ \cancel{a_0}}^{x_*} \frac{dx}{\hbar} \sqrt{2m(V(x) - \cancel{E})} = \frac{\sqrt{2m}}{\hbar} \int_0^{x_*} dx \sqrt{V_0 - q\mathcal{E}x}$$

$$= \frac{\sqrt{2m}}{\hbar} \sqrt{q\mathcal{E}} \int_0^{x_*} dx \sqrt{x_* - x} \simeq \frac{2}{3} \frac{\sqrt{2mq\mathcal{E}}}{\hbar} x_*^{3/2}$$

$$= \frac{2}{3} \frac{\sqrt{2mV_0^3}}{\hbar} \cdot \frac{1}{q\mathcal{E}}$$

$$\tau \sim \# e^{\left(\frac{2}{3} \cdots \cdot \frac{1}{\mathcal{E}}\right)}$$

non-perturbative!