PHYS 5210

Graduate Classical Mechanics

Fall 2022

Lecture 18

Effective field theory of a solid

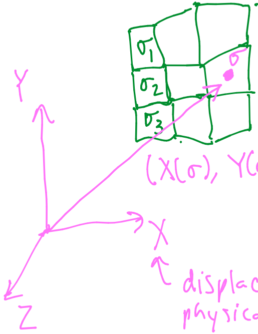
October 3

1 What are the degrees of freedom in a solid? What are the symmetries?

Solid as a phase of matter?

- (semi)-rigid structure
- global translation/rotation

symmetries under $\mathcal{L}$



} mass dm

continuous

$(X(\sigma), Y(\sigma), Z(\sigma))$

label for each element

displacement/ physical coord.

Lec 12-14: $x \to \sigma$

~~CHOICE:~~ this class

- $\sigma^I(X_i)$: Eulerian

field coord

- $X_i(\sigma^I)$: Lagrangian

Lec 12-14; modern research

→ easier to write down $\mathcal{L}$.

rotational symmetries:

$X_i \to R_{ij} X_j$

$R \in SO(3)$

$\sigma^I \to Q^{IJ} \sigma^J$

$Q \in SO(3)$

translation symmetry:

$t \to t + \varepsilon_t$ OR $X_i \to X_i + \varepsilon_i$ OR $\sigma^I \to \sigma^I + \varepsilon^I$

2 Explain why a solid spontaneously breaks symmetry.

Solid, in equilibrium, spontaneously break symmetry!

$\sigma^I = X_i \delta_i^I$

X-transl: $\sigma^I \to \sigma^I$ but $X_i \to X_i + \varepsilon_i$

- solutions to E-L equation / state of system has lower symmetry

What symmetries does state preserve?

- $t \to t + \varepsilon_t$
- $X_i \to X_i + \varepsilon_i$ AND $\sigma^I \to \sigma^I - \varepsilon_i \delta_i^I$

  $\sigma^I \to (X_i + \cancel{\varepsilon_i})\delta_i^I - \cancel{\varepsilon_i \delta_i^I}$

- preserving rotations

  $\sigma^I \to R^{IJ} \sigma^J ; \; X_i \to (R^{-1})_{ij} X_j$

$X \to X + \varepsilon$

$\sigma \to \sigma - \varepsilon$

$(\sigma^1, \sigma^2, \sigma^3)$

$\boxed{\vec{\sigma}}$

$\sigma = (0,0,0)$

3 Write down the Lagrangian for a solid.

Look for invariant building blocks!

[$\mathcal{L}$ has ALL symmetries: σ trans/rot, X trans/rot, $t \to t + \varepsilon_t$]

σ^I needs derivatives [$\sigma^I \to \sigma^I + \varepsilon^I$]:

$\to \partial_i \sigma^I \partial_i \sigma^I$ is invariant!

$\partial_t \sigma^I \partial_t \sigma^I$ is invariant!

$\partial_i \sigma^I \partial_j \sigma^I \partial_j \sigma^J \partial_i \sigma^J$ also OK, but we'll neglect...

$$\mathcal{L} = \tilde{A}(\partial_t \sigma^I \partial_t \sigma^I, \partial_i \sigma^I \partial_i \sigma^I)$$

$$\to A\, \partial_t \sigma^I \partial_t \sigma^I - F_1\, \partial_i \sigma^I \partial_i \sigma^I - F_2\, \partial_i \sigma^I \partial_i \sigma^I \partial_j \sigma^J \partial_j \sigma^J$$

Solid, we will need to expand around ansatz:

$$\sigma^I = X_i \delta_i^I + \phi^I(X)$$

infinitesimal

$\boxed{4}$ Expand out the Lagrangian to quadratic order in ϕ^I.

$$\rightarrow \underline{A\, \partial_t \sigma^I \partial_t \sigma^I} - \underline{F_1\, \partial_i \sigma^I \partial_i \sigma^I - F_2 (\partial_i \sigma^I \partial_i \sigma^I)(\partial_j \sigma^J \partial_j \sigma^J)}$$

Said, we will need to expand around ansatz:

$$\sigma^I = X_i \delta_i^I + \underbrace{\phi^I(X)}_{\text{infinitesimal}}$$

$$A\, \partial_t \phi^I \partial_t \phi^I \; \cancel{-3F_1} - \cancel{2F_1 \partial_i \phi^i} - F_1 \partial_i \phi^I \partial_i \phi^I - \overset{2\cdot 3}{6} F_2 \partial_i \phi^I \partial_i \phi^I - F_2 \cdot 4\, \partial_i \phi^i \partial_j \phi^j$$

$$\partial_i \sigma^I \partial_i \sigma^I = (\delta_i^I + \partial_i \phi^I)(\delta_i^I + \partial_i \phi^I)$$

$$= 3 + 2\, \partial_i \phi^i + \partial_i \phi^I \partial_i \phi^I$$

$$\mathcal{L} \approx A\, \partial_t \phi^I \partial_t \phi^I - (F_1 + 6F_2) \underbrace{\partial_i \phi^j \partial_i \phi^j}_{\partial_i \phi_j \cdot \partial_i \phi_j} - 4F_2\, \partial_i \phi^i \partial_j \phi^j$$