PHYS 5210
Graduate Classical Mechanics
Fall 2023

Lecture 1
The principle of least action

August 28

Office Hrs: 12-1 Wed 4-5 Th

Duane F629

2023 Fall Term HW/ exam

PHYS 5210, Fall 2023

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- Zoom
- My Course Materials

recorded

PHYS 5210
Graduate Classical Mechanics
Fall 2023

General Course Information:

- Syllabus (course specific)
- Syllabus (university required statements)
- Holistic grading scheme
- Slack channel invite link

docs linked

Lectures:

Lectures will generally take place in person in G2B47. If the lecture is marked "Zoom", then it will be synchronously delivered but only on Zoom. If the lecture is marked with an alternate time, it will also be given only on Zoom. Lectures will generally be recorded and accessible via the Zoom Canvas plug-in. Recommended reading is listed below.

Lecture 1	August 28	The principle of least action	JS 3.1.1
Lecture 2	August 30	Invariant building blocks	N/A

Zoom only announced here

José & Saletan

HW due on Fri nights (Sun) (expect ≈ 13 HWs)

total points of A

0, 3, 6, 9, 12, 15

(15) **A:** Let us begin by studying the case where the rod is not under stress.

A1. Explain why curves $y(x) = c$ (with c constant) are a global translation of the rod.

A2. Explain why $y(x) = bx$ (for infinitesimal b) can be thought of as a global rotation of the rod.

} subparts = suggested parts

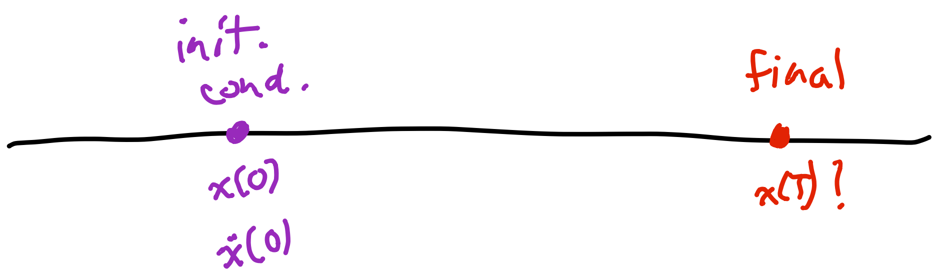
each HW graded out of 100

115 ~ 125 total points

3 HW drop/extensions (48 hrs)

Final grade: 60% HW 40% exam

The old way of mechanics:



Newton: $F = ma = m\dfrac{d^2x}{dt^2}$

Solve ODE.

$\ddot{x} = \dfrac{d^2x}{dt^2}$ $\left(\dot{x} = \dfrac{dx}{dt}\right)$

physics!

microscopic...

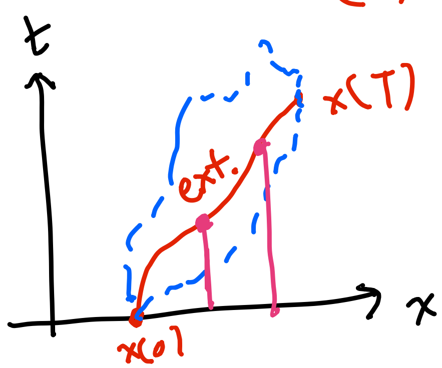
$F(x, \dot{x}, t)$

Lagrangian mechanics: start w/ postulate

Principle of least Action: there's functional (action, S) is extremum (minimum) on "physical trajectories"

functional = "function of functions" (real number)

$$S[x(t)] : \left\{ \begin{array}{c} \text{trajectories } x(t) \\ \text{w/ fix} \\ x(0) \text{ \& } x(T) \end{array} \right\} \longrightarrow \mathbb{R}$$

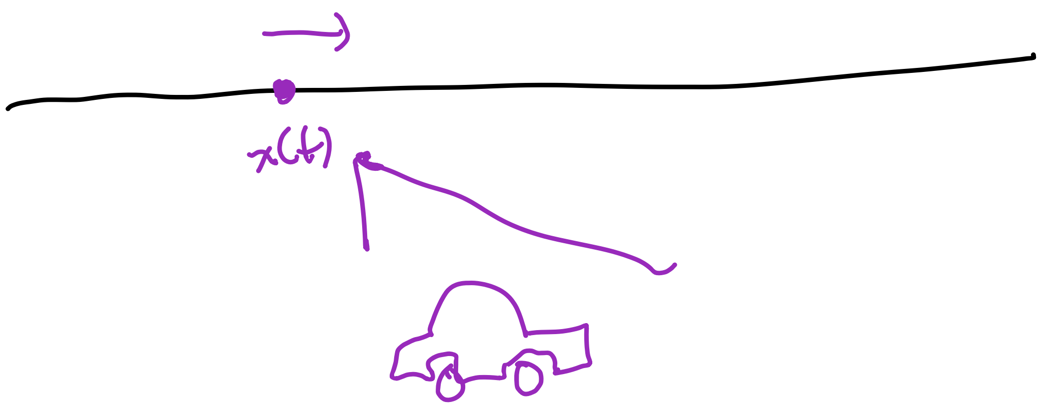


dumb choice for S:

$$S[x(t)] = \begin{cases} 1 & x \text{ not phys.} \\ 0 & \text{phys.} \end{cases}$$

Claim: "guess" form of S based on $\underbrace{\text{physical principles}}$.

effective (field) theory $\left\{ \begin{array}{l} \text{locality, symmetry,} \\ \text{simple/ perturbative} \end{array} \right.$



Today: locality in time constrains S.

$F = ma$, but F depends on x, $\dot{x}$ but not $x(t+1)$

Use P.O.L.A.: assume S "differentiable"

calculus: find extrema by $\frac{\partial S}{\partial x_1} = \cdots = \frac{\partial S}{\partial x_n} = 0$.

For now: $S[x(t)] \to S(\cancel{x(0)}^{\text{fixed}}, x_1 = x(\Delta t), x_2 = x(2\Delta t), \ldots, x_n = x(t - \Delta t), \underbrace{\cancel{x(t)}}_{\text{fixed}})$

POLA $\to \frac{\partial S}{\partial x_1} = \cdots = \frac{\partial S}{\partial x_n} = 0$.

each one of these constraints $\rightsquigarrow$ local EOM.

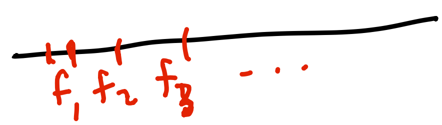
Now include generic terms in $S = \cdots + a x_1 x_n + \cdots$

$$\frac{\partial S}{\partial x_1} = 0 = \underbrace{\cdots}_{f_1'(x_1)} + a \underbrace{x_n}_{x(t-\Delta t)} \sim \text{"}F = ma\text{" at } t = \Delta t.$$

forbidden by locality...

So... $S = \underline{f_1(x_1)} + \underline{f_2(x_1, x_2)} + \underline{f_3(x_2, x_3)} + \cdots$

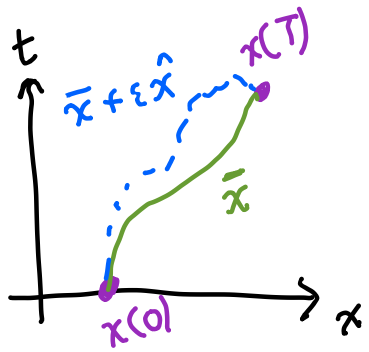
Take continuum limit for t...:



$$S[x(t)] = \int_0^T dt\, \underbrace{L}_{\text{Lagrangian}}(x, \dot{x}, \ddot{x}, \ldots, t)$$

Derive (Euler-Lagrange) equations of motions

Claim: POLA $\leftrightarrow$ $\underbrace{\frac{\delta S}{\delta x(t)}}_{\text{functional der.}} = 0$



$$\left.\frac{dS[\bar{x}+\varepsilon\hat{x}]}{d\varepsilon}\right|_{\varepsilon=0} = \int_0^T dt\ \hat{x}(t)\frac{\delta S}{\delta x(t)}$$

$$\frac{dS}{d\varepsilon} = \int_0^T dt\ \frac{d}{d\varepsilon} L(\bar{x}+\varepsilon\hat{x}, \dot{\bar{x}}+\varepsilon\dot{\hat{x}})$$

$$= \int_0^T dt \left[\frac{\partial L}{\partial x}\hat{x} + \underbrace{\frac{\partial L}{\partial \dot{x}}\dot{\hat{x}}}_{\cancel{\left.\hat{x}\frac{\partial L}{\partial \dot{x}}\right|_0^T} - \int_0^T \hat{x}\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) dt}\right]$$

$\hat{x}(0) = \hat{x}(T) = 0.$

$$= \int_0^T dt\ \hat{x}(t)\left[\frac{\partial L}{\partial x} - \frac{d}{dt}\frac{\partial L}{\partial \dot{x}}\right]\Bigg|_{\bar{x}}$$

POLA: $\frac{\delta S}{\delta x(t)} = 0$ (at all t):

physical trajectories obey $\frac{\partial L}{\partial x} - \frac{d}{dt}\left(\frac{\partial L}{\partial \dot{x}}\right) = 0.$