PHYS 5210
Graduate Classical Mechanics
Fall 2024

Lecture 1
Principle of least action

August 26

Office hours: Mon 12-1 & Fri 3-4

HW due on Tues - submit Canvas
- graded out of 100 pts. ~115-125 avail. (last problem = hard!)
- 3 extensions ($\leq$ 2 day)

Final grade: 60% HW
40% exam

2024 Fall Term

- **Home**
- Assignments
- Grades
- People
- Zoom

if traveling/sick: class recorded

PHYS 5210, Fall 2024

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optional reading:
JS - José & Saletan

General Course Documents:

- Syllabus (course specific)
- Syllabus (university required statements)
- Course Slack (invitation link)
- Some typed up notes

noted here

Lectures:

Lecture 1	August 26	Principle of least action	JS 3.1.1, notes 1-2
Lecture 2	August 28	Symmetry groups	notes 3.1-3.3

PDF of lecture

Homework:

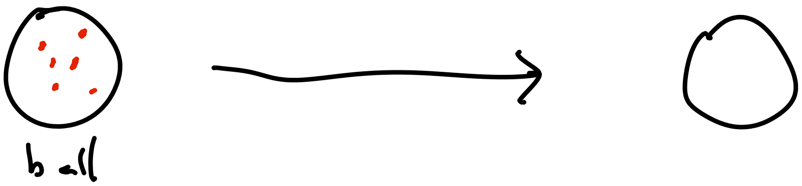
Homework 1	due September 10	Solutions	Grade Distribution
Homework 2	due September 17	Solutions	Grade Distribution

Exam:

Exam	due December 16	Solutions	Grade Distribution

World = complex

$\sim 10^{26}$ atoms



ball

$F_{ball} = m a_{ball}$

$F_1 = m a_1$

$F_2 = m a_2$

$\vdots$

Life goal: model as simply as possible

$\downarrow$

Build an effective theory!

1) what system / state?
2) regime of validity?
3) identify important DOF
4) use physical principles (locality & symmetry) to constrain possible.

$\sim 10^{26}$ atoms (gas / solid?)

$\Delta t \gg t_{vib}$

all terms allowed in EOM probably there

Steps 1-3 make sense w/ $F = ma$.

Step 4 does not.

- subtle symmetry (relativity?)
- in step 3, DoF may not live $\mathbb{R}^n$ $\nearrow (x_1, \dots, x_n)$

  finding generalized $F = ma$ hard?

Build an effective theory $\rightarrow$ new perspective.

$\downarrow$

Lagrangian mechanics!

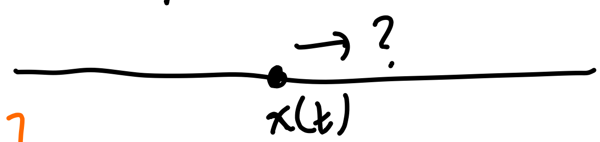
Postulate: Principle of Least Action:

There exists a functional (action, S), which is extremal on physical trajectories on configuration space (w/ fixed endpoints).

Goal: find EOM ($F=ma$)

Configuration space: denoted X. Set of all physically distinct "configurations"...

Today: $X=\mathbb{R}$ ← real line



Functional: $S: \left\{ \begin{array}{l} \text{smooth trajectory} \\ x(t): [0,T] \to X \\ \text{w/ fixed endpts} \end{array} \right\} \to \mathbb{R}$

↳ $S[x(t)]$ is a real number.

$$S_{troll}[x] = \begin{cases} 1 & x \text{ not physical} \\ 0 & x \text{ physical} \end{cases} \quad \to \min_{x_{phys}} \| x - x_{phys} \| \quad !!$$

Problem: physics is predictive science.
Deduce S before we know physical trajectory

Claim: locality in time

↳ $ma = m\dfrac{d^2x}{dt^2} = f(x)$

$= \displaystyle\int_{-\infty}^{t} ds \; g(x(t); x(s))$ or worse

implies

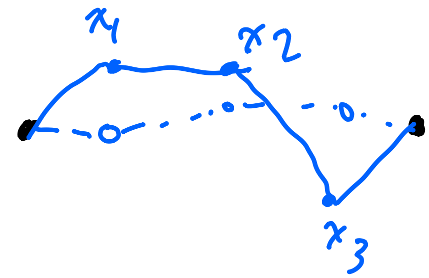
$\dot{x} = \dfrac{dx}{dt}$

$$S[x(t)] = \int_0^T dt \; \underbrace{L(x, \dot{x}, \ddot{x}, \ldots, t)}_{\text{the Lagrangian.}}$$

[add dissipation → fluctuation-dissipation thm]

Justify claim:

simplify to



Multivar calculus:

Extrema of $S(x_1, x_2, x_3, \ldots)$ are $\underbrace{\frac{\partial S}{\partial x_1} = \frac{\partial S}{\partial x_2} = \cdots = 0}$.

if each $\frac{\partial S}{\partial x_3}$ was discretatization of EOM near t_3.

Locality: $\frac{\partial S}{\partial x_3} = f_3(\underbrace{x_2, x_3, x_4}_{\approx \text{ finite order } t\text{-der.}}) = 0$

$\hookrightarrow$ $S = F_3(x_2, x_3, x_4) + S_{rest}(x_1, x_2, \ldots, x_4 \ldots)$

re-do arg. for each point: relabel

$$S = \sum_t F_t(x_{t-1}, x_t, x_{t+1}) = \sum_t \Delta t \cdot L_t(x_{t-1}, x \ldots)$$

continuum limit: $S = \int_0^T dt \; L(x, \dot{x}, \ldots, t)$

How to extremize functional S?

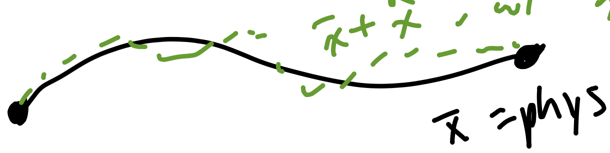
Goal: $\frac{\delta S}{\delta x(t)} = 0$

funct. der. $\left(\sim \frac{\partial S}{\partial x_j}\right)$

phys arb. / same endpts

Idea: $\frac{\delta S}{\delta x(t)} = 0$ at each t if $\frac{d}{d\varepsilon} S[\bar{x} + \varepsilon \hat{x}]\Big|_{\varepsilon=0} = 0$.

$\bar{x} + \hat{x}$, w/ $\hat{x}(0) = \hat{x}(T) = 0$.



If $S = \int dt\, L(x, \dot{x}, t)$ [for simplicity]

$$\frac{d}{d\varepsilon} S[\bar{x} + \varepsilon \hat{x}]\Big|_{\varepsilon=0} = \int_0^T dt\, \frac{d}{d\varepsilon} L(\bar{x} + \varepsilon\hat{x}, \dot{\bar{x}} + \varepsilon\dot{\hat{x}}, t)$$

Goal:

$$= \int_0^T dt \left[\hat{x} \frac{\partial L}{\partial x} + \dot{\hat{x}} \frac{\partial L}{\partial \dot{x}} \right] \to \int_0^T dt\, \hat{x}(t) \frac{\delta S}{\delta x(t)}$$

$$= \int_0^T dt \left[\hat{x} \underbrace{\left(\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}} \right)}_{} \right] + \cancel{\hat{x} \frac{\partial L}{\partial \dot{x}}\Big|_0^T}$$

$\sim \sum \hat{x}_n \frac{\partial S}{\partial x_n} =$ dir der.

$\hat{x}(0) = \hat{x}(T) = 0$

$$\frac{\delta S}{\delta x} = \underbrace{\frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}}}_{}$$

$$0 = \frac{\partial L}{\partial x} - \frac{d}{dt} \frac{\partial L}{\partial \dot{x}}$$ is Euler-Lagrange equation.

$$\frac{d}{dt} \frac{\partial L}{\partial \dot{x}} = \dot{x} \frac{\partial^2 L}{\partial x \partial \dot{x}} + \cdots + \frac{\partial^2 L}{\partial t \partial \dot{x}}$$