α Decay

In this problem, we will explore the decay of a heavy nucleus using the WKB approximation. A heavy nucleus typically decays via α decay, where the nucleus loses two protons and two neutrons. We will model this crudely by the following model in 1D. We will model the α particle by a particle of mass $m \approx 7 \times 10^{-27}$ kg, sitting inside of the potential

$$V(x) = \begin{cases} \infty & x < 0 \\ 0 & 0 < x < a \\ \dfrac{2Ze^2}{4\pi\epsilon_0 x} & x > a \end{cases} .$$

If the particle has an energy E when sitting inside the well, we want to know how long it will take for the particle to tunnel out of the nucleus.

(a) Approximate that $a = Z^\nu a_0$, where $a_0 \approx 3 \times 10^{-15}$ m. What should the exponent ν be?

(b) Use the WKB approximation to estimate the lifetime τ of the atom – the time it will take for the α particle to decay out of the nucleus.

(c) Typically, the ejected α particles have a speed of about 1.5×10^7 m/s. Plug in for the value $Z = 90$ (thorium), and the values for m and a_0, and estimate a numerical value for the lifetime in years. Half-lives of thorium atoms are either 10^3 or 10^{10} years, roughly speaking – is the model accurate for any of the isotopes?

The wild variations in the half-lives of nuclei suggest that simplistic models such as this are not capturing much of the internal dynamics.