Enzyme Reaction Dominated by Tunneling

One of the big debates right now in molecular biophysics is whether or not quantum effects are relevant in a large number of important processes, such as photosynthesis. However, it was demonstrated experimentally in 2006 that there exists an enzymatic reaction in which quantum tunneling is dominant over thermal fluctuations.

The reaction in question is essentially an enzyme catalysis in which a proton (mass $m \approx 1.7 \times 10^{-27}$ kg) is transferred between molecules across a gap of length[1] $a \approx 0.06$ nm with a free energy barrier of about $G \approx 20 k_{\mathrm{B}} T$. The temperature should be taken to be $T = 300$ K.

In this problem we will use scaling arguments to determine whether or not this claim is feasible. In both quantum tunneling and thermal fluctuations, the time scale for a particle to hop over a barrier is of the form

$$\text{rate} = \frac{\text{exponential factor}}{\tau}$$

where τ is some time scale. In this problem you may assume that τ is the same for both the classical and quantum processes, so just focus on the exponential factor.

(a) What is the rate (up to τ) for the classical process?

(b) What is the rate (up to τ) for the quantum process, using the WKB approximation? Use simple scaling arguments.

(c) What is the ratio of the quantum rate to the classical rate? Plug in for the given numbers and comment whether or not for the given reaction we should have expected quantum tunneling to dominate.

[1] Note that this length is shorter than the hydrogen-hydrogen chemical bond.