Symmetry of Ammonia

The ammonia (NH_3) molecule can be roughly approximated by placing a nitrogen atom at the center of a tetrahedron, with 3 hydrogen atoms at 3 of the corners of the tetrahedron.

(a) Explain why the symmetry group of ammonia, about the central nitrogen atom, is S_3.

(b) The "natural" representation of S_3 corresponds to the matrices whose actions on basis vectors are $\sigma \cdot e_i = e_{\sigma(i)}$. Explain why this must be a reducible representation.

(c) Find an explicit decomposition of the representation above into 2 irreps, by describing how the σ matrices described in the previous part may be transformed into block-diagonal matrices.

In general, the task of part (c) is quite gross, but this is an example where it is at least plausible to do by hand, so you can see how it is done.