Hot Circuit Elements

In this problem, we will explore two simple examples of when temperature effects alter the behavior of a circuit element. In each problem, for simplicity assume that the resistance of a resistor is linearly proportional to temperature T:

$$R = \alpha T.$$

At very large temperatures this is basically true.

Let us begin with a discussion of a light bulb. A light bulb can be thought of as a resistor which is meant to run at very hot temperatures. The dissipated energy $P = I^2R$ is used to emit light, and the hot body should radiate away this light via blackbody radiation.

(a) Conclude that the power output of a lightbulb is related to the applied voltage as $P \sim V^{8/5}$.

Next, we consider "blowing a fuse". The fuse is a circuit element which will burn up if one tries to apply too large a current. Let us assume that the fuse is a resistor, as before, and that the temperature of the resistor is given this time by

$$T = T_0 + \kappa P,$$

where P is the power dissipated. Note that here, the energy is not carried away by blackbody radiation.

(b) Show that the circuit element will "blow" (in the sense that the temperatures become so large that they should melt the material) as I approaches a critical value I_c. Find an expression for I_c.