Evolution and Fitness

Consider a population of individuals with varying genomes. Let the fraction of individuals with genome i be

$$x_i = \frac{N_i}{N}$$

where N_i is the total number of people of each genome and

$$N = \sum_i N_i.$$

Now, let f_i be the fitness of genome i: i.e., the rate at which genome i reproduces. Let q_{ij} be the probability that a genome j gives birth to a genome i.

(a) Using the equations of population growth, and taking into account the probabilities of mutation, write down the first order ODEs describing the evolution of $N_i(t)$, on average. Note that these equations are linear.

(b) Show that

$$\dot{x}_i = \sum_j q_{ij} f_j x_j - x_i \sum_j f_j x_j$$

In reality, it is the dynamics of $x_i(t)$ that we are interested in when studying evolution. However, these equations are nonlinear and in general the number of genomes may be very large, so in general we might think that this equation will be quite hard to solve! Remember though, that we have the linear differential equations for $N_i(t)$!

(c) Solve the linear differential equation you found, and use this to determine the large t behavior of the x_i equations. In particular, show that, assuming that all $q_{ij} > 0$, there is a unique fixed point of the x_i equations which is globally stable, and determine how to find an explicit expression for this fixed point.

(d) The linear equation you solved involves a matrix. What is the physical interpretation of the largest eigenvalue of the matrix?