Seignorage and Hyperinflation

One of the more "devious" things that governments try to do is to print more money to simply pay the bills: this practice is called **seignorage**. Denote by M by the money supply of the economy, and P the price level. Note that we can relate the inflation rate, π, to the change in the money supply as

$$\pi = \frac{\dot{M}}{M}.$$

Empirical relations suggest furthermore that

$$\frac{M}{P} = M_0 \mathrm{e}^{-b\pi}$$

for constants M_0 and b. The rate of money raised through seignorage is given by

$$S = \frac{\dot{M}}{M}\frac{M}{P} = M_0 \pi \mathrm{e}^{-b\pi}$$

Let's try and figure out how much money the government can get through seignorage.

(a) Determine the value of π for which S is a maximum, and find this value of S.

(b) Empirical data suggests $M_0 \sim 0.09$, $b \sim 0.4$. In these units, we are measuring the rate of inflation per year. How much inflation per year is there if the government is optimizing the amount of money raised through seignorage? Does this seem like something industrialized governments are doing today?

The model we have so far however does not account for why a government would cause hyperinflation: a situation where the price level will approximately increase by a factor of 1.5 each month! One way we can account for this to put dynamics into the model. Denote $M/P = m$. We found earlier that $m = M_0 \mathrm{e}^{-b\pi}$ in equilibrium. It is natural to expect $\log m$ to undergo relaxational dynamics to this equilibrium:

$$\frac{\dot{m}}{m} = \eta \left[\log M_0 - b\pi - \log m\right],$$

where η is some rate. We also know that

$$\dot{m} = S - \pi m,$$

i.e. the growth of the real money supply is related to the seignorage minus inflationary reduction. These dynamics can lead to hyperinflationary scenarios. Let's see how:

(c) Combine these two equations to remove π.

(d) Qualitatively describe the dynamics of $m(t)$. Show that the dynamics undergoes a bifurcation at a critical value of seignorage, S_c and find an expression for S_c in terms of appropriate parameters.

(e) Conclude that for $S > S_\mathrm{c}$ hyperinflation is likely to ensue (you probably should resort to a "practical", not mathematical, argument).