Superradiant Dynamics

Superradiance corresponds to a collective excitation of coherent photons. More precisely, if we have N atoms in a box, in a superradiant transition the intensity of the light in the box will jump from $\sim N$ to $\sim N^2$. Although this is a quantum phenomenon, in this problem, we will study semiclassical dynamics of a superradiant system.

We will model the N atoms by a spin vector $\mathbf{S} = (S_x, S_y, S_z)$, and a complex number ψ which corresponds to the coherent photonic excitation. When $\psi = 0$, the intensity in the box is $\sim N$; when $\psi \neq 0$, the intensity in the box is $\sim N^2$. We will frequently denote $S_\pm = S_x \pm \mathrm{i}S_y$. One can argue that a very general class of dynamics is given by the set of equations

$$\begin{aligned}
\dot{S}_\pm &= \pm\mathrm{i}\left[\left(\omega_0 + U|\psi|^2\right)S_\pm + 2g\left(\psi + \overline{\psi}\right)S_z\right], \\
\dot{S}_z &= \mathrm{i}g\left(\psi + \overline{\psi}\right)\left(S_- - S_+\right), \\
\dot{\psi} &= -\mathrm{i}g\left(S_- + S_+\right) - \left(\kappa + \mathrm{i}\left(\omega + US_z\right)\right)\psi.
\end{aligned}$$

In this problem, we assume $\omega_0 > 0$. Physically, the parameter κ represents a "leakage" rate for photons out of the box, ω, ω_0, U, g.

(a) Show that $|\mathbf{S}|^2$ is constant. You should set $|\mathbf{S}| = N/2$ for the remainder of the problem.

(b) Show that there are three classes of fixed points, described as follows. The first is "trivial" and corresponds to $\psi = S_x = S_y = 0$. The non-trivial, superradiant fixed points, fall into two classes: one where $S_y = 0$, and one where $U < 0$, and $S_y \neq 0$. We will refer to these phases as A and B, respectively.

(c) Begin by studying phase A. For what ranges of parameters can this phase exist? Does the problem appear symmetric under $S_z \to -S_z$? Determine the locations of possible fixed points.

(d) Study the onset of superradiant instability in phase A, starting from $S_z = -N/2$. (Note that the constraint of part (a) should help to reduce the number of independent variables to consider!) What is the time scale associated with the onset of superradiance (when you are close to the transition)? Be careful to consider all possible cases.