

Business Cycle

Modern economies frequently experience what is called a **business cycle** where many things, such as wages, unemployment rates, GDP, etc., fluctuate over time (often not in phase with each other). Much to the annoyance of citizens and politicians, it seems very hard to avoid these fluctuations. This problem asks the question: is it possible that business cycles might be *fundamental* macroeconomic phenomena? The classical answer to this question is no: classical macroeconomics suggests that the economy is inherently stable, and that a business cycle is the result of something such as a supply shock (e.g., the world oil supply drastically falls), a natural disaster, an ill-advised government policy, etc. In this problem, we will dispute this classical assertion.

Let the variable Y denote the rate of growth of income, and K denote the rate of growth of the amount of capital¹ in the economy. We postulate that the dynamics of Y and K are given by

$$\begin{aligned}\dot{Y} &= \alpha(I - S), \\ \dot{K} &= \zeta I - \delta K,\end{aligned}$$

where I relates to the investment rate, S to the savings rate, α relates the rate of growth of the growth rate of income to the factors which adjust incomes (namely, whether money is invested into the economy, or is simply saved) δ is the rate at which capital depreciates (or breaks down), and ζ to the rate at which investment can generate capital.

We still need to describe what I and S are. Reasonable assumptions are that

$$I = I_{\max} \frac{Y^2}{Y^2 + Y_0^2} - \beta K.$$

This is because the investment rate usually is capped at some reasonable level: there are not an infinite number of worthwhile investments that the investors will be aware of, etc. We also want $\beta > 0$.² A simple postulate for S is that

$$S = \gamma Y$$

for $\gamma < 1$.

(a) Show that the system of equations can be nondimensionalized to

$$\begin{aligned}\dot{Y} &= a \left[\frac{Y^2}{Y^2 + 1} - cY - bK \right], \\ \dot{K} &= \frac{Y^2}{Y^2 + 1} - (1 + b)K.\end{aligned}$$

(b) Explain the intuitive economic meaning of a , b and c .

(c) Draw the phase plane plot with the nullclines of Y and K .

¹Capital refers to things like factories, equipment and machinery, etc. There can also be human capital: education, etc.

²As the technology gets better, it becomes less necessary to build new stuff!

(d) Explain why, for the model to make any economic sense, we need to take

$$c < \frac{1}{2(1+b)}.$$

(e) Show that there is a critical a , a_c , such that for $a > a_c$ the model is a nonlinear oscillator. Find the value of a_c .

(f) Sketch $Y(t)$ and $K(t)$ assuming $a > a_c$. Comment on your results: do they make economic sense?

In conclusion, we have shown that when incomes respond too rapidly compared to the changes in capital, we should *expect* natural business cycles to occur. We might expect that this is a reasonable assumption, in fact: it's easy for an employer to slash paychecks, e.g., but not so easy to just build new factories and equipment.

This model is of course not a definitive proof of inherent instabilities in the economy, but it should make you call into question the assumptions of classical economics. It may very well be the assumption that markets will always tend towards equilibrium is not a very good one, and that instead, business cycles are *natural* and not the result of either market or government inefficiencies.