Cheyne-Stokes Respiration

A respiratory ailment known as Cheyne-Stokes respiration is characterized by breathing patterns which are periodic in intensity. It is proposed that this ailment results from a breathing pattern where the volume of air intake of each breath is too sensitive to the concentration of oxygen in the bloodstream. The goal of this problem is to propose a simple model and check if it would confirm this hypothesis.

Let $V(c)$ be the volume of a breath as a function of oxygen concentration. We assume that V is monotonically increasing. A simple, but nontrivial, model for the rate of change of oxygen concentration is given by

$$\dot{c} = \frac{c_0 V_0 - c(t)V(c(t-T))}{\tau}.$$

Here c_0 is some sort of optimal oxygen concentration, $V_0 = V(c_0)$, τ is the time scale of adaptation of oxygen concentration, and T is a time delay due to the fact that the body cannot instantaneously determine the oxygen concentration, but must rely on an estimate from an earlier time.

(a) Show that $c = c_0$ is the only fixed point of the dynamics.

(b) Linearize the dynamics around the fixed point assuming exponential time dependence. Assuming that the perturbation scales as $e^{\lambda t}$, find an equation for λ. Express your answer in terms of $a = V'(c_0)$.

(c) We are interested biologically in the stability of the fixed point. Determine the stability of the fixed point as a function of T. In particular show that for $0 \leq T < T_1$ the fixed point is stable, and find the value of T_1.

(d) Was the biological proposition at the beginning of the problem correct?

(e) What is the period of oscillations at the onset of instability?