Diamagnetism

If we have a collection of n identical atoms per unit volume placed in an external magnetic field, what is the collective response of the material? If the response is dominated by the fact that the electron "orbits" around the nuclei are altered, then the material is called *diamagnetic*. This problem develops a simple classical model for understanding diamagnetism.

To begin, consider a lone atom. Let the mass of an electron be m, and its charge be $-e$; suppose it orbits around a nucleus of effective charge Ze in an orbit of radius a with velocity v.

(a) What is the magnetic moment μ of a single atom, in terms of v, a, e, m and fundamental constants?

Now suppose we turn on an external magnetic field, such that after some later time the strength of the magnetic field is B. The orientation of the external magnetic field is in the same direction as the atom's magnetic moment. As we turn on the field, we will certainly induce an electric field.

(b) Draw all forces acting on the electron in orbit, at a time when $\mathrm{d}B/\mathrm{d}t \neq 0$.

(c) Prove that the radius of orbit is constant as the magnetic field is turned on.

(d) Find an expression for the change in magnetic moment $\Delta\mu$, after the field is at its final strength B.

In general, we can define the magnetization of the medium to be $M = n\Delta\mu$. We define the magnetic susceptibility of a material to be the dimensionless constant χ such that

$$B = \mu_0 \left(1 + \frac{1}{\chi}\right) M.$$

For any reasonable material, $\chi \ll 1$.

(e) Find an approximate formula for χ given the answer to part (d). Is χ positive or negative for a diamagnetic material?