Holograms

Roughly speaking, photographic material is one where the transmittance is proportional to the intensity of light that shines upon it while it is developing: i.e.,

$$t_{\text{photo}} \approx t_0 + \alpha I + \cdots$$

Obviously, this is only valid for a moderate period of time; afterwards, the photo is developed and is not changing. In this problem, we use the scalar wave approximation, and let $\psi^*\psi = I$.

Photographs, however, lose half of the information about the incident wave – the phase. A *hologram* is a slab of material which will record both the amplitude and the phase of an incident wave.[1] This is done by a trick, which was Gabor's clever intuition: use a reference wave which will add to the incident wave, so that the total wave incident on the photographic medium is $\psi = \psi_{\text{obj}} + \psi_{\text{ref}}$. Typically, $|\psi_{\text{obj}}| \ll |\psi_{\text{ref}}|$.

(a) Write out an expression for t_{photo}, assuming a reference wave is present and taking into account the approximation.

A piece of photographic material is in the $z = 0$ plane. Suppose that the incident wave is a spherical source originating from $(x_\text{o}, y_\text{o}, z_\text{o})$, and the reference wave is a spherical source originating from $(x_\text{r}, y_\text{r}, z_\text{r})$. The photographic sheet is allowed to develop and then a plane wave is incident on it, propagating in the positive z direction.

(b) Show that it will appear to an observer as if a plane wave is originating from a point $(x_\text{i}, y_\text{i}, z_\text{i})$. Find an expression for these three coordinates.

(c) Use the result of part (b) to find the *magnification* of an image along the x and y directions, M_{xy}, and along the z direction, M_z.

The key result of this problem is as follows – for a normal photograph, because I is real, the image will appear to originate from the $z = 0$ plane – it will appear two-dimensional. But for a hologram, $z_\text{i} \neq 0$, in general; the image will truly appear 3–D; only by capturing information about the phase of the wave can we observe this effect!

[1] Holograms were invented by Dennis Gabor in 1948; he won the Nobel Prize in Physics in 1971 for this.