Optical Molasses

The Nobel Prize in physics in 1997 was given for work involving "optical molasses", which is the cooling of atoms by focused laser beams. In this problem, we will explore a simple model for how this can work.

Consider an atom of mass m, moving with some velocity v. If a photon of energy $\hbar\omega$ (as observed in the atom's reference frame) collides with the atom, the atom will absorb the energy of the photon, and transition into an excited quantum mechanical state. It will then emit a photon of the same frequency (in it's frame) and transition back to the ground state.

Now, imagine that this atom is moving with velocity v. Far to the right of the atom, a laser is emitting photons, traveling towards the atom, at frequency ω_{L}.

(a) What is the required frequency ω such that the atom may absorb a photon from the laser?

(b) In the laboratory frame, what is the momentum of the atom after it absorbs the photon?

(c) Now, the atom will typically quickly decay back to the ground state, and in doing so it will emit a photon. Half of the time this photon will move to the left, and half of the time this photon will move to the right. In the atom's frame, the photon has energy $\hbar\omega_0$ and momentum $\pm\hbar\omega_0/c$. In the laboratory frame, determine the momentum of the atom if the photon moves leftwards, and also if it moves rightwards.

(d) What is the average change in momentum of the atom after the absorption/emission process?

Now, imagine that we place this atom in between two lasers, both emitting photons at frequency ω_{L}. We also need to be a bit more careful: since this is a quantum mechanical process, it turns out that the above description is not quite correct. What we should instead say is that the rate with which an atom will interact with a photon of frequency ω is given by

$$\text{number of interactions per unit time} = \frac{\Gamma\Omega^2}{\Omega^2 + \dfrac{\Gamma^2}{4} + (\omega-\omega_0)^2}.$$

This function is peaked around ω_0, but for ω "close to" ω_0, the number of interactions is non-negligible.

(e) Assuming that $v \ll c$, use the results from the previous parts to show that the force applied to the atom is given by

$$F \approx \frac{4\Gamma\Omega^2\hbar\omega_{\mathrm{L}}}{c^2\left(\Omega^2 + \dfrac{\Gamma^2}{4} + (\omega_{\mathrm{L}}-\omega_0)^2\right)}(\omega_{\mathrm{L}}-\omega_0)v$$

Comment on the answer: depending on ω_{L}, what happens?

(f) If the velocity of the atom is v_0 at time $t = 0$, find the velocity $v(t)$. Show that the answer depends qualitatively on $\omega_{\mathrm{L}} - \omega_0$.

Now, using statistical physics, one can show that the temperature of a gas of atoms is related to $T \sim \langle v^2\rangle$, roughly speaking, with $\langle v^2\rangle$ a thermal average of the velocities. Therefore, by slowing down $\langle v^2\rangle$ to a very small number, using the trick above, one could cool the gas of atoms. Although there are quantum constraints on how low the temperatures can get by this method, it does provide a very useful way to look at cold atoms, which are an important area of modern physics.