# Rectangular Dielectric Waveguide

Consider an infinite plane slab of (non-magnetic) dielectric material, with index of refraction $n$ and thickness $2a$. Choose rectangular coordinates such that the slab is located at $|z| \le a$, as depicted below:

In this problem, we will consider waves which are uniform in the $x$-direction, and which propagate along the $y$-direction, and explore the properties of a waveguide made up of a dielectric slab, instead of a metal tube.

(a) Write down Maxwell's equations, assuming that the fields all take the form $f(z)\mathrm{e}^{\mathrm{i}(ky-\omega t)}$. Remember, you must consider the fields in *all space*.

(b) What fields are non-zero in a TE mode (scalar $\psi = B_y$) and in a TM mode (scalar $\psi = E_y$)? What are the boundary conditions for each mode?

(c) Explain why, in both modes,

$$\frac{\omega^2 n^2}{c^2} > k^2 > \frac{\omega^2}{c^2}.$$

(d) Let $\alpha^2 = k^2a^2 - \omega^2a^2/c^2$ and $\beta^2 = n^2\omega^2a^2/c^2 - k^2a^2$. Show that, for TE modes,

$$\tan\beta = -\frac{\beta}{\alpha} \text{ or } \frac{\alpha}{\beta}.$$

Plot this equation as a function of $\beta$: comment on where each mode lies.

(e) Instead considering TM modes, show that

$$\tan\beta = -\frac{\beta}{\alpha n^2} \text{ or } \frac{\alpha n^2}{\beta}.$$

Plot and comment, as in part (d).

(f) Do TE or TM modes tend to propagate with a lower frequency? Is the answer what you expect?

This is a very simple model for a fiber optic cable (essentially this problem, but in a cylindrical geometry, where the mathematics becomes much more irritating). Fiber optic cables are critical for modern technology, as they are a very cheap and efficient way to transmit electromagnetic waves over very long distances with very little attenuation of the signal.