Aberrations in Optical Systems

Often times when constructing an optical system, we are faced with the reality that the imaging is imperfect due to the presence of *aberrations* in the pupil. An aberration does not cause the pupil to obtain a distorted shape; instead, it causes the ray passing through each point on the pupil to be distorted by a different phase. Thus, we can account for aberrations by letting our transmittance be

$$t(\mathbf{x}) = P(\mathbf{x})\mathrm{e}^{\mathrm{i}kW(\mathbf{x})}$$

where $W(\mathbf{x})$ is a function which accounts for the aberrations and $P(\mathbf{x})$ is the unaberrated pupil function. Let z be the distance to the image plane of the optical system and k be the wave number of the incoherent light.

(a) Let $\mathcal{H}_0$ be the OTF of the pupil with transmittance $P(\mathbf{x})$, and $\mathcal{H}'$ be the OTF of the pupil with transmittance $t(\mathbf{x})$ defined above (with aberrations). Show that

$$\mathcal{H}'(\mathbf{q}) = \frac{\displaystyle\iint_{A(\mathbf{q})} \mathrm{e}^{\mathrm{i}k[W(\mathbf{x}+z\mathbf{q}/2k)-W(\mathbf{x}-z\mathbf{q}/2k)]}\mathrm{d}^2\mathbf{x}}{\displaystyle\iint_{A(\mathbf{0})} \mathrm{d}^2\mathbf{x}}$$

Recall that $A(\mathbf{q})$ corresponds to the overlap area between $P(\mathbf{x}+\lambda z\mathbf{q}/2)$ and $P(\mathbf{x}-\lambda z\mathbf{q}/2)$.

(b) Show that $|\mathcal{H}'| \le |\mathcal{H}_0|$: i.e., aberrations can never increase the resolution at a spatial frequency.

A square aperture of width b serves as the exit pupil of an optical system whose image plane is located a distance $z+\zeta$ away from the exit pupil (remember, the imaging is done at z). This aberration is therefore a simple focusing error (e.g., someone put in a lens of the wrong focal length!).

(c) By writing down the function $W(\mathbf{x})$ exactly, show that if

$$\eta = kb^2\left(\frac{1}{z} - \frac{1}{z+\zeta}\right),$$

$$\mathcal{H}'(\mathbf{q}) = \mathcal{H}_0(\mathbf{q})\cdot \mathrm{j}_0\left(\eta\frac{|q_x|}{2q_0}\left(1-\frac{|q_x|}{2q_0}\right)\right)\mathrm{j}_0\left(\eta\frac{|q_y|}{2q_0}\left(1-\frac{|q_y|}{2q_0}\right)\right)$$

where $q_0 = kb/2z$.

(d) Let $q_y = 0$. Plot $\mathcal{H}'$ vs. q_x for $\eta = 0,\ 0.25,\ 0.5,\ 1$. What is the critical value of η, η_c, for which $\mathcal{H}'$ can take on *negative* values?

(e) If $\mathcal{H}'(\mathbf{q}) < 0$, areas of high intensity become areas of low intensity and vice versa. Can you give an explanation for this?