Damped Waves in a Plasma

A plasma consists of a roughly neutral ionized gas, with immobile ions and n_0 electrons per unit volume of charge $-e$ and mass m. Suppose that each electron, as it moves about, is damped by a force $\mathbf{F}_\mathrm{d} = -m\nu\mathbf{v}$, due to collisions with the ions. Electromagnetic plane waves are propagating through the plasma, such that $\mathbf{k} \cdot \mathbf{E} = 0$.

(a) Using Maxwell's equations and Newton's Law for the electrons, show that the dispersion relation is

$$c^2k^2 = \omega^2 - \frac{{\omega_\mathrm{p}}^2}{1 + \mathrm{i}\nu/\omega}$$

where ${\omega_\mathrm{p}}^2 = n_0 e^2/m\epsilon_0$ is the plasma frequency.

(b) If ω is real and $\nu \ll \omega$, show that the "skin depth" δ of the plasma is given by

$$\delta = \frac{1}{\mathrm{Im}(k)} \approx \frac{2c\omega\sqrt{\omega^2 - {\omega_\mathrm{p}}^2}}{\nu {\omega_\mathrm{p}}^2}.$$

(c) A proposed method for providing power to humans on Earth is to direct the energy of the sun through the ionosphere via a powerful laser. The ionosphere is a region of the Earth's atmosphere which can be thought of as a weak plasma. Note that an approximate formula for the collision damping coefficient ν is $\nu = \kappa n_\mathrm{n}$, where n_n is the density of neutral (not ionized) atoms per unit volume. If $\kappa = 10^{14}\ \mathrm{m}^3/\mathrm{s}$, the laser emits light with wavelength of 0.3 m, the thickness of the ionosphere is 100 km, the fraction of ionized atoms in the ionosphere is 10^{-5} and there are 10^{16} atoms per m^3 in the ionosphere, find the ratio of the intensity of the beam after exiting the plasma to the intensity before entering. How reasonable is this proposal?