Distance to an Astronomical Object

In this problem, we will find a simple model for estimating the distance to a far off astronomical object, such as a star or a nebula. As in general with astronomy, we find data by looking at the light (electromagnetic radiation) emitted by the object. A simple approximation is that the radiated electromagnetic power P, generated by the interaction of the ions and electrons in a volume V, is given by

$$P = \alpha V n_+ n_-.$$

where n_+ is the ion number density, and n_- is the electron number density.

(a) Suppose that there are N electrons and N ions, each spread uniformly throughout the astronomical object, which we will take to be a sphere of radius r. Find the expression for P in terms of α, r and N.

(b) Now, to collect the light from this distant object here on Earth, we set up a telescope with a camera. If we aim the camera at this object, we can gain two pieces of information very simply: firstly, the intensity I of the light collected from the object, and secondly, the angular size ϕ of the object in the camera. Show that the distance d to the object is

$$d = \frac{K}{\phi^{3/5} I^{1/5}}$$

and find the coefficient K in terms of α and N.

(c) A naïve guess would have that $d \sim I^{-1/2}$. Explain why this is not the case, intuitively.

Unfortunately, since N and r may be unknown in general, this is not the best way to determine distances in actual modern astronomy. However, I hope this problem has given you a rough idea of some of the optical principles involved.