

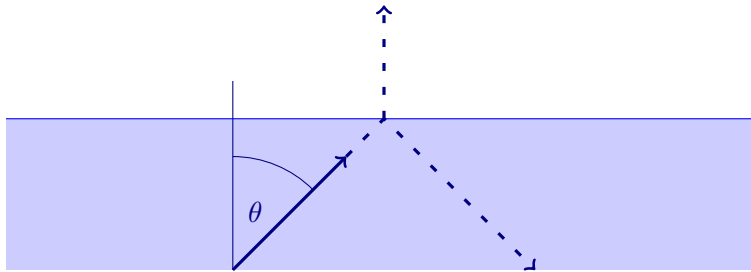
## Evanescent Waves

There is a critical angle,  $\theta_c$ , at the boundary between a dielectric and (for convenience) the vacuum, such that at  $\theta \geq \theta_c$  “all” of the wave is reflected. What does that precisely mean? It does *not* mean that the fields in the other medium are zero, as that would clearly violate Maxwell’s equations and the associated boundary conditions.

In this problem, we will assume an incident wave of the following form: the electric field is oriented along the  $x$ -direction (parallel to the boundary). Let the dielectric medium have index of refraction  $n$ . We assume that  $n \sin \theta > 1$ . Let

$$k = \frac{n\omega}{c},$$

$$\kappa = \frac{\sqrt{n^2 \sin^2 \theta - 1}\omega}{c}.$$



(a) Show that the electric fields of the incident, reflected, and transmitted waves are exactly given by

$$E_i = E_0 e^{i(ky \sin \theta + kz \cos \theta - \omega t)},$$

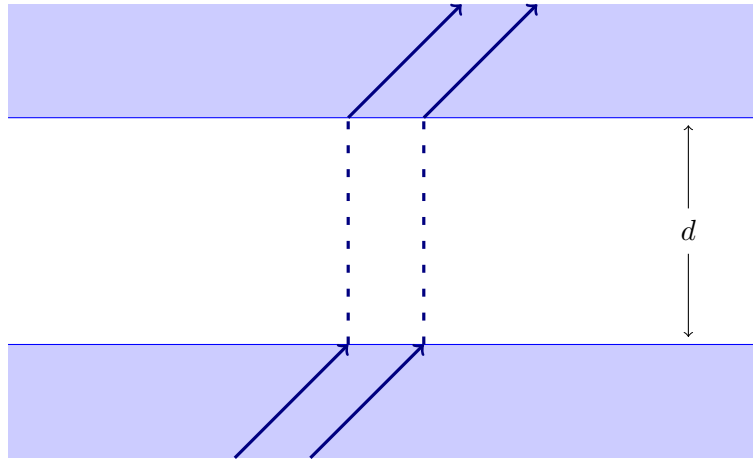
$$E_r = E_0 \frac{n \cos \theta - i\sqrt{n^2 \sin^2 \theta - 1}}{n \cos \theta + i\sqrt{n^2 \sin^2 \theta - 1}} e^{i(ky \sin \theta - kz \cos \theta - \omega t)},$$

$$E_t = E_0 \frac{2n \cos \theta}{n \cos \theta + i\sqrt{n^2 \sin^2 \theta - 1}} e^{i(k \sin \theta y - \omega t)} e^{-\kappa z}.$$

The wave in vacuum is called an *evanescent wave*. It does not propagate outwards and carry energy/momentum, it simply exists to satisfy the boundary conditions.

Nonetheless, evanescent waves can have real, noticeable effects. Suppose we have two slabs of index of refraction  $n$ , separated by a distance  $d$ . If a wave is incident in the lower slab (with  $n \sin \theta > 1$ ) a small wave will also propagate identically (with a phase shift, though) in the upper slab. Without an understanding of Maxwell’s equations, such a phenomenon would be incredible (it can’t occur just from geometric optics), but with Maxwell’s equations, the appearance of such a phenomenon isn’t very

surprising. This “jumping” of the wave across the vacuum is called the *Goos-Hänchen effect*.



If the initial beam of light has intensity  $I_0$ , and the measured intensity in the light across the gap is  $I$ , the following equation holds:

$$\log \frac{I}{I_0} = \alpha - 2\kappa d.$$

This is the Goos-Hänchen law, and a plot verifying this law can be generated in a modern lab with fairly basic equipment. You may assume that  $\kappa d \gg 1$ .

- (b) Verify the Goos-Hänchen law; in particular, find the empirical constant  $\alpha$  in terms of the parameter  $z = \kappa/k \sin \theta$ .