Fabry-Perot Interferometer

A plane sheet of an excellently conducting metal has width L, and is placed in vacuum.[1] The conductivity of the metal is σ and its skin depth is δ; assume that $\delta \gg L$. For convenience, let

$$\gamma = \frac{\sigma L}{\epsilon_0 c}.$$

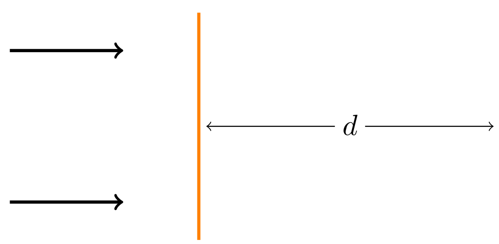
(a) Show that the presence of the sheet of metal can be replaced by altering the boundary conditions on $\mathbf{H}$.

(b) A normally incident electromagnetic wave of wavelength λ, such that $\lambda \gg l$, is incident on the plane sheet. Show that the reflection and transmission coefficients are

$$R = \frac{\gamma^2}{(2+\gamma)^2},$$
$$T = \frac{4}{(2+\gamma)^2}.$$

(c) Clearly, $R + T < 1$. What does this imply? Verify global conservation of energy.

We can construct a *Fabry-Perot interferometer* by placing two such sheets of metal a distance d apart. This device will, roughly speaking, allow only certain wavelengths to pass through, while reflecting all others.



(d) Show that the transmission coefficient for the combined system for a normally incident electromagnetic wave of wave number k is given by

$$T = \frac{1}{(1+\gamma)^2 + \left(\frac{\gamma}{2}\right)^2 \left(1 + \frac{\gamma}{2}\right)^2 \sin^2(kd)}.$$

(e) Calculate γ for a sheet of aluminum ($\sigma = 4 \times 10^7\ (\Omega \cdot \mathrm{m})^{-1}$) 2 nm thick.

(f) Let $x = kd$. Plot $T(x)$ for this value of γ and comment on its shape.

(g) Numerically find the half-width x_h of the transmission function[2] for this value of γ. How large is x_h compared to π?

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[1] Thanks to Alexander Fetter for giving me this problem.
[2] If $x = x_\mathrm{m}$ is a maximum of $T(x)$, then $T(x_\mathrm{m} \pm x_\mathrm{h}) = \frac{1}{2} T(x_\mathrm{m})$.