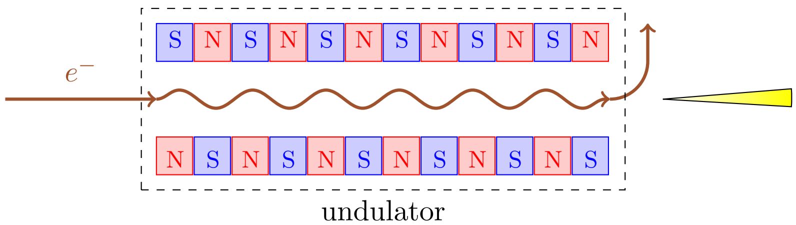
Free Electron Laser

The **free electron laser** (FEL) is a device which creates a highly directed beam of photons within a fairly narrow range of frequencies. A basic schematic for a FEL consists of the following apparatus: electrons traverse a region of rapidly spatially oscillating magnets (called an *undulator*), causing them to emit highly directed radiation in the forward direction.[1]



To model the FEL, consider an electron of mass m and charge $-e$ initially moving at velocity v_0. Let frame $\bar{S}$ move with this velocity so that initially, the electron is stationary in $\bar{S}$. For simplicity, we will assume that the acceleration is not transverse to the initial motion, and is given by

$$\bar{a}(\bar{t}) = \begin{cases} a_0 \cos \omega_0 \bar{t}, & |t| < \tau \\ 0 & |t| > \tau \end{cases}$$

in $\bar{S}$, for $\tau = N\pi/\omega_0$.

(a) Show that

$$\frac{\mathrm{d}^2 E}{\mathrm{d}\Omega\, \mathrm{d}\omega} = \frac{e^2 {a_0}^2}{4\pi^3 {\omega_0}^2} \frac{\sin^2\theta}{(1 - v_0 \cos\theta)^4} f_N\left(\frac{\omega}{\omega_0}(1 - v_0\cos\theta)\right)$$

for an electron undergoing this acceleration, assuming that $a_0 \ll v_0\omega_0$, and

$$f_N(x) = \frac{N\pi}{2}\frac{x}{x^2 - 1}\sin(N\pi x).$$

(b) Plot $f_N(x)$ for $N = 1$ and $N = 20$, and comment on the results. At which value of N would we want our laser to operate?

(c) Plot the angular distribution at $\omega = \omega_{\max}$, the frequency at which the expression in part (a) is a maximum for θ_0, for $v_0 = 0.99$.

[1]Thanks to Alexander Fetter for giving me this problem.