# Helicon Waves

Suppose we have a very high conductivity metal, such as copper, placed a very large external magnetic field,

$$\mathbf{B} = B\hat{\mathbf{z}}.$$

Let $n$ be the density of electrons, which have mass $m$ and charge $-e$. The electrons obey an equation of motion

$$m\frac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = -e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) - \frac{m}{\tau}\mathbf{v}.$$

Assume that an electric field wave of the form $\mathbf{E} = \mathbf{E}_0 \mathrm{e}^{\mathrm{i}(\mathbf{k}\cdot\mathbf{r} - \omega t)}$ is propagating through the metal. Express answers in terms of $\omega_\mathrm{c}$, $\sigma_0$, and $\omega_\mathrm{p}$, where

$$\sigma_0 = \frac{ne^2\tau}{m},$$
$$\omega_\mathrm{c} = \frac{eB}{m},$$
$$\omega_\mathrm{p} = \sqrt{\frac{ne^2}{m\epsilon_0}}.$$

(a) Find the resistivity tensor $\rho_{ij}$, such that $E_i = \rho_{ij} J_j$.

(b) Invert $\rho_{ij}$ to find the conductivity tensor, $\sigma_{ij}$, such that $J_i = \sigma_{ij} E_j$.

(c) If the copper is at temperature 7.7 K, then the following numerical data holds: $e = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg, $n = 8.5 \times 10^{28}$ m$^{-3}$, $\sigma_0 = 5.0 \times 10^{8}$ $(\Omega \cdot \mathrm{m})^{-1}$. Verify that in this situation, $\omega_\mathrm{c}\tau \gg 1$.

(d) Simplify $\sigma_{ij}$ in the limit $\omega\tau \ll 1 \ll \omega_\mathrm{c}\tau$.

(e) Use Maxwell's equations to show that, in this limit, circularly polarized waves of electric fields can propagate without decay at frequencies given by

$$\omega = \pm\alpha k^2,$$

and find the constant $\alpha$.

These waves are called "helicon" waves; this phenomenon is sometimes called the Hall effect as well, although this is usually referring to the classic experiment used to determine the sign on the charge carriers in a material. Note that the magnetic fields must be huge in order for this effect to become noticeable.