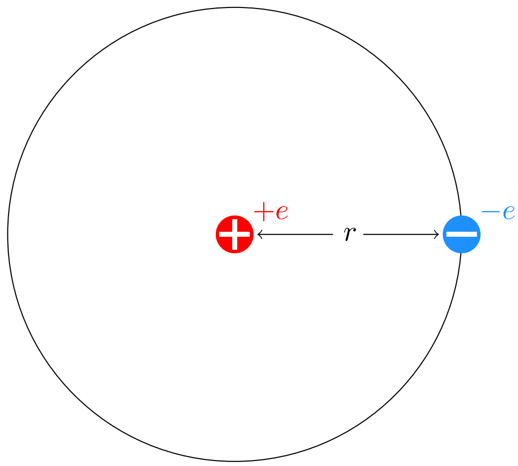
Bohr Model of the Hydrogen Atom

In 1915, Niels Bohr presented a simple model of a hydrogen atom, consisting of a single proton of charge $+e$, orbited by an electron of mass m and charge $-e$. Suppose the electron has a radius of orbit r.



(a) What is the energy E of the electron in orbit?

Bohr made one further assumption – that the angular momentum of the electron was somehow constrained to integer values of a constant, $\hbar$.[1] Therefore,

$$L = n\hbar, \quad n = 1, 2, 3, \ldots$$

(b) Show that the allowed energies of the electron are given by

$$E = -\frac{E_1}{n^2}$$

where E_1 is the ground state ($n = 1$) energy, and find an expression for E_1 in terms of the parameters of the problem. This is the lowest possible energy obtainable by an electron in the hydrogen atom.

(c) What is the energy of the ground state, measured in eV (electron volts)?

As you likely know from high school physics and/or chemistry courses, the Bohr model predicts the energies of the hydrogen atom incredibly accurately: in fact, not until the introduction of relativistic quantum mechanics was the result of part (b) found to be incorrect (ignoring perturbations due to spin effects). While ultimately, the model used to justify the result of part (b) is fundamentally wrong, it is still a pleasing result which can be obtained without fancy electromagnetic techniques.

One of the things which ultimately led physicists to suspect that this picture was completely wrong is the fact that accelerating charges *radiate*: i.e., they lose energy to electromagnetic radiation. The rate at which a charge q with acceleration a loses energy is given by Larmor's formula:

$$P = \frac{q^2 a^2}{6\pi\epsilon_0 c^3},$$

where c is the speed of light.

[1] $\hbar$ experimentally is given by $h/2\pi$, where h is Planck's constant, a constant which had been found earlier in experiments such as the photoelectric effect.

(d) Use this result to find the lifetime τ of the hydrogen atom in seconds, assuming that it starts off in the ground state at $E = -E_1$: i.e., how long does it take for an electron to spiral into the proton. Assume that the motion of the electron is nonrelativistic, and that the radius of the electron does not appreciably change in a given orbit. Is the answer surprising?