## The Child-Langmuir Law

A vacuum diode consists of electrons of charge -e and mass m flowing between two metal plates, separated by a distance d. Electrons flow from the anode at potential  $\varphi = 0$  to the cathode at potential  $\varphi = \varphi_0$ . Assume a steady-state (time-independent) potential  $\varphi(x)$ , dependent only on x, the distance from the anode.



- (a) Why is the current density J a constant function of x?
- (b) Relate the electron speed v(x), the potential  $\varphi(x)$  and the charge density  $\rho(x)$  to obtain an ODE for  $\varphi(x)$ .
- (c) Solve the ODE from part (b) under the proper boundary conditions to show that

$$J = k\varphi_0^{3/2},$$

and find the constant of proportionality k. This equation is called the **Child-Langmuir law**.