The Liquid Drop Model

A nucleus consists of A nucleons, Z of which are protons. The mass of a proton is m_{p}; the mass of a neutron is m_{n}. In addition, interactions within the nucleus give it a nonzero binding energy E_{B}.[1]

(a) What is the mass m of the nucleus?

The liquid drop model refers to a semi-empirical formula for the binding energy:

$$E_{\mathrm{B}} = a_1 A - a_2 A^{2/3} - a_3 \frac{Z^2}{A^{1/3}} - a_4 \frac{(A-2Z)^2}{A^{4/3}} + a_5 f_{\mathrm{pair}}(A,Z),$$

where $a_1, a_2, \ldots, a_5$ are positive empirical constants with units of energy and

$$f_{\mathrm{pair}}(A,Z) = \begin{cases} 1 & A,\ Z \text{ even} \\ 0 & (A+Z) \text{ odd} \\ -1 & A,\ Z \text{ odd} \end{cases}.$$

The nucleus is modeled as a mixed fluid of nucleons, each contributing an equal volume v, assembled into a uniform sphere. Each proton carries a net charge $+e$, and the neutrons carry no charge.

In this problem, we will use basic physics to estimate these empirical parameters, first theoretically, and then "experimentally" by using the masses of various nuclei.

(b) A uniformly charged sphere has electrostatic potential energy. What a_i refers to the electrostatic energy of the nucleus? Find a theoretical expression for this a_i.

(c) Nucleons interact attractively via the strong force. As a rough model of this strong force, we presume that the energy per unit volume of a strongly interacting fluid is given by $-\gamma$. Which a_i does this correspond to? Find a theoretical expression for this a_i.

(d) The energy of a collection of N fermions goes like $KN^{5/3}$; assume that K is the same for both protons and neutrons. For nucleons, it turns out that $KA = \kappa$ is (very roughly) a universal constant. Now, we know that when $A = 2Z$ the total fermionic (kinetic) energy is minimized; when $A - 2Z$ is non-zero (but small) this energy will be increased. Show that two a_i correspond to the two lowest order terms in the expansion about this minimum of fermionic kinetic energy.

(e) Protons and neutrons are both spin-half particles; for every pair that exists, there is so-called *spin-spin coupling* that leads to a negative energy. A nucleus is most stable when all of the nucleons are coupled by spin to another nucleon. Roughly speaking, which a_i should this correspond to?

The model is ultimately a semi-empirical one: we hardly expect any of the formulas found in parts (b) through (d) to be anything better than an order of magnitude estimate, if that. An alternative empirical approach to estimating the a_i constants is to simply look at numerical data from actual nuclei.

(f) Use the tables below to find numerical estimates for all five a_i.

This approach ultimately is quite imperfect as well (you may be able to arrive at many "different" answers for part (f) just by picking different nuclei!): the liquid drop model is simply not complex enough to do a good job of finding the mass of a complex nucleus.

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[1] If you want to break the nucleus into its components, you need to provide an energy E_{B}.

nucleon	mass (10^{-27} kg)
p	1.672
n	1.675

isotope	mass (m_{p})
${}^{14}_{7}\mathrm{N}$	14.0031
${}^{15}_{7}\mathrm{N}$	15.0001
${}^{16}_{8}\mathrm{O}$	15.9949
${}^{238}_{92}\mathrm{U}$	238.0508
${}^{238}_{93}\mathrm{Np}$	238.0509
${}^{238}_{93}\mathrm{Np}$	239.0529
${}^{238}_{94}\mathrm{Pu}$	238.0496