

Breakdown, Atmospheric Electricity and Lightning

Lightning strikes occur when a large enough potential energy difference is accumulated between the ground and the clouds to ionize air molecules. Once ionized, electric charges can rapidly flow in the electric fields to help neutralize the charge difference. In this problem, we will try and understand some of the qualitative features of lightning.

To begin, we need to understand a phenomenon called breakdown: when we apply a large enough electric field ($|\mathbf{E}| \geq E_c$), we notice that the air begins to substantially ionize. Typical values for air are $E_c \approx 3 \times 10^6$ V/m. How does this spontaneous ionization occur? Let's consider we start with one free electron in the air (for example, suppose a cosmic ray ionizes a single atom). This electron will be accelerated in the electric field $\mathbf{E}$ and after traveling for some distance, will collide with a neutral atom. Now, suppose that this electron has enough energy to, on average, ionize the atom it collides with. Then we would have 2 free electrons that will get accelerated again and ionize a new atom. If this process can continue like this, we will have an avalanche of ionization events, which culminates in macroscopic ionization of the air.

Actually, the process is a bit more complicated than that. The key subtlety is that when an electron is moving very fast, it experiences a large frictional force due to complicated electromagnetic interactions with the neighboring air. After a complicated calculation, one finds that the typical scale of this frictional force is

$$F = \frac{ne^4}{4\pi\epsilon_0^2 U_i}$$

where U_i is the ionization energy of an air molecule, e is the charge on the electron, and n is the density of air molecules.

- (a) Verify that F has the dimensions of a force.
- (b) A typical electron has energy of order U_i if it is being accelerated during breakdown. If the electron mass is about 10^{-30} kg, about how fast is the electron moving?
- (c) Using $e = 1.6 \times 10^{-19}$ C, $U_i \approx 3 \times 10^{-18}$ J, that the density of air is about 1 kg/m^3 , and that the proton mass is about 1.6×10^{-27} kg, estimate the electric field strength E_c at which electrons can be accelerated and overcome this frictional force. Compare to the experimental value.

Now, let's talk about thunderstorms. Although you may not feel it, there are actually fairly substantial atmospheric electric fields. The height of a typical cloud is about 5 km, and the voltage difference between the ground and the cloud can get close to 400 kV.

- (c) What is the average electric field in between the cloud and the ground? Compare to E_c and comment.
- (d) If your body was a perfect insulator, what would be the voltage between your feet and the top of your head?¹

¹In reality, your body acts as a fairly good conductor, so you do not actually have such a large voltage drop across your body!

During a thunderstorm, however, the local electric fields can get quite large, and can in fact exceed E_c . The precise mechanism for how is *still unknown*, amusingly enough! Regardless, let us take on faith that a lightning bolt can in fact get started. The lightning bolt will correspond to a thin column of ionized air in which the electrons can flow to balance the charge difference between the cloud and the ground.

- (e) Estimate the duration of a lightning bolt.²
- (f) Estimate the amount of charge transferred during a lightning strike if the current flow is about 30 kA. How many electrons does this correspond to.
- (g) Estimate the amount of energy transferred during a lightning strike.

Although lightning is a phenomenon we are all familiar with, it is still strangely mysterious in many ways. For example, the model of the first few parts of this problem was proposed only in 1992!

²How long would it take an electron to travel from the ground to the cloud?