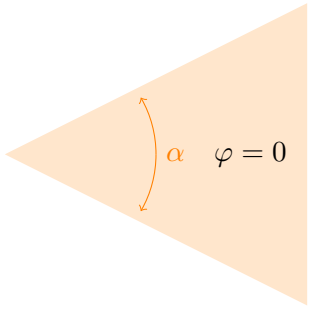
Electrostatics near a Corner

Consider a conducting metal, at potential $\varphi = 0$, in the shape of a circular wedge of angle α. Assume there are no charges in the remainder of the space.



(a) Use separation of variables to compute the most arbitrary solution to Laplace's equation that is continuous at all points in the space.

(b) Compute the electric field near the origin, assuming an arbitrary potential. In general, what happens near the origin? Show that the electric fields are unbounded if $\alpha < \pi$.

This result has very practical applications: one of them is to explain how something like a lightning rod works. The basic idea is this: if the lightning rod has a sharp corner ($\alpha < \pi$), then the electric fields near that corner will be unbounded. However, when the electric fields get strong enough, they can actually ionize the air near them, and this can lead to a lightning strike which will reduce the external potential to something less intense.