1+1D Dirac Fermion Confinement

Consider a charged Dirac fermion of mass m and charge e in 1+1D, placed in an electric potential $A^t(x,t) \equiv \varphi(x)$. Assume $A^x(x,t) = 0$. Recall that a Dirac fermion in 1+1D can be described by a 2 component spinor: e.g.,

$$\Psi(x,t) \equiv \begin{pmatrix} \chi(x,t) \\ \omega(x,t) \end{pmatrix}.$$

(a) Show that with an appropriate choice of γ matrices, an energy eigenstate of the Dirac equation with energy E satisfies the equation

$$E\begin{pmatrix} \chi \\ \omega \end{pmatrix} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\frac{\partial}{\partial x}\begin{pmatrix} \chi \\ \omega \end{pmatrix} - m\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} \chi \\ \omega \end{pmatrix} + e\varphi(x)\begin{pmatrix} \chi \\ \omega \end{pmatrix}$$

Assume that $\varphi(x) = \varphi(-x)$. Under only this assumption, we will be able to prove a very strict theorem on confinement.

(b) Show that $\langle x \rangle = 0$.

(c) Use the identity

$$\int_{-\infty}^{\infty} \mathrm{d}x\; x\chi\frac{\mathrm{d}\chi}{\mathrm{d}x} = -\frac{1}{2}\int_{-\infty}^{\infty} \mathrm{d}x\; \chi^2$$

to show that

$$\left|\int_{-\infty}^{\infty} \mathrm{d}x\;\; x\chi\omega\right| = \frac{1}{4m}.$$

(d) Conclude that

$$\langle x^2 \rangle - \langle x \rangle^2 \geq \frac{1}{4m^2}.$$