Yukawa Theory RG Flow

The Lagrangian for Yukawa theory is

$$\mathcal{L} = -\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}M^2\phi^2 - \mathrm{i}\bar{\psi}\not{\partial}\psi - m\bar{\psi}\psi - \frac{\lambda}{24}\phi^4 - g\phi\bar{\psi}\psi.$$

Work in $d = 3 + 1$ spacetime dimensions.

(a) Show that the 1-loop β functions for the coupling constants are

$$\beta(g) = \frac{5g^3}{16\pi^2},$$
$$\beta(\lambda) = \frac{3\lambda^2 + 8\lambda g^2 - 48g^4}{16\pi^2}.$$

To study the RG flows of Yukawa theory, define

$$\alpha \equiv \frac{\lambda}{g^2}.$$

(b) Using the results of part (a), show that

$$g = g_0 \log\left|\frac{\alpha - \alpha_+^*}{\alpha - \alpha_-^*}\right|^\nu$$

describes the RG flow trajectory of the coupling constant g, and find the constants $\alpha_\pm^*$ and ν.

(c) Sketch the RG flows of (λ, g^2). Which fixed points are stable, and which are unstable? What happens at high energy, and at low energy?