Fibonacci Numbers

Define $a_0 = a_1 = 1$, and then define

$$a_n = a_{n-1} + a_{n-2}$$

for $n \geq 2$. This generates the sequence of **Fibonacci numbers**, which surprisingly appear quite often in biology!

Define a vector

$$\mathbf{a}_n = \begin{pmatrix} a_n \\ a_{n-1} \end{pmatrix}.$$

(a) Find the matrix T such that

$$\mathbf{a}_{n+1} = T\mathbf{a}_n.$$

(b) Find the eigenvalues and eigenvectors of T.

The matrix formalism allows one to exactly answer questions which would be very difficult by other means.

(c) Find a closed form expression for a_n as a function of n only.

(d) Show that

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{1+\sqrt{5}}{2}.$$

This number is called the **golden ratio**. It was deeply appreciated by the ancient Greeks and appears frequently in their architecture.