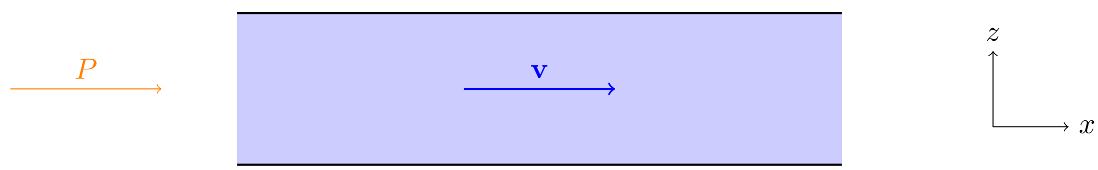
Resistance with Slip Length

We usually talk about the no-slip boundary condition in fluid mechanics: at a wall, $\mathbf{v} = \mathbf{0}$. Sometimes this may not be true, however. In nano fluid systems, it appears as though the boundary conditions may sometimes be

$$\mathbf{v}_\perp = -\lambda(\hat{\mathbf{n}} \cdot \nabla)\mathbf{v}_\perp,$$

where $\mathbf{v}_\perp$ is the tangential velocity to the boundary.

Let's see what this does in a simple example. Consider the flow down a very long channel between two parallel plates, separated by a distance h, with a pressure gradient of P/L:



(a) Find the velocity $v_x(z)$ by solving the Navier-Stokes equations with the slip length boundary condition.

One of the things we care about is the so-called resistance of this channel. For us, we will want to work with the resistance per unit width of the channel, which can be defined as

$$R = \frac{Q}{P},$$

where P is the pressure applied, and Q is the volume flow rate per depth of the channel. Note that we will find $R \sim L$, the total length of the channel.

(b) Show that the presence of a slip length reduces the resistance by

$$R(\lambda) = \frac{R(0)}{1 + 6\dfrac{\lambda}{h}}.$$

(c) Some experiments with water suggest that λ may be about 50 nm. Do you think this effect is observable if h is about 1 mm? How about 1 μm?