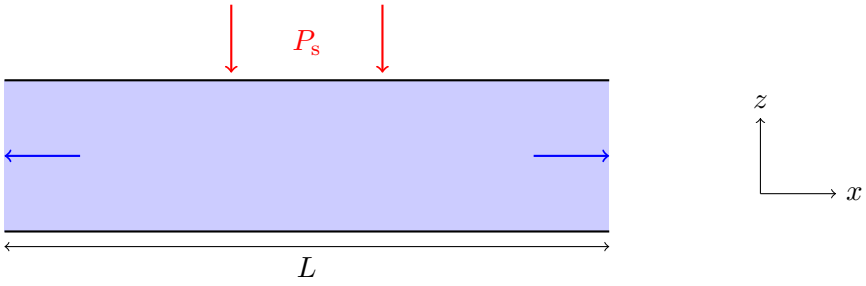
Squeeze Flow

In this problem we will consider a situation called "squeeze flow", where we apply an average constant pressure P_s to the top plate of a parallel plate, with viscous fluid of viscosity η in between. This will make the height $h(t)$ between the plates a decreasing function of time. We assume that the parallel plates are infinite in the y direction, and have a length $L \gg h(t)$ in the x direction. Set $x = 0$ to the center of the plates.



(a) Use mass conservation to show that the volume flow rate per unit width (in the y direction), Q, is get by

$$Q = -\dot{h}x.$$

(b) Now, approximate that $\dot{h}$ is slow enough, and h is small enough, that the approximation that we have a Poiseuille flow is reasonable. Use this approximation to find $v_x(x, z)$.

(c) Use a conservation law to find $v_z(x, z)$.

(d) Given v_x and v_z, determine the pressure $P(x, z)$ for $2|x| < L$. Normalize by using that $P(x, 0) = 0$ for $2|x| > L$.

(e) Now, find $\dot{h}$, given that the average pressure on the top of the top plate is P_s.

(f) Determine the function $h(t)$. Show that for large times t:

$$h(t) \approx \sqrt{\frac{\eta L^2}{2P_s t}}.$$