Viscosity of a Suspension

Often times, very small solid particles are "suspended" in a much larger fluid – we call such systems suspensions. In this problem, we will determine through qualitative arguments how the suspension alters the viscosity of the fluid as a whole. Let the viscosity of the much larger fluid be η_{f}, and its mass density be ρ_{f}. For simplicity, assume that the suspension consists of spherical particles of radius a and mass density ρ_{s}. To compute the viscosity of a fluid is to subject the fluid to a flow of the form

$$\mathbf{v} = \alpha y \hat{\mathbf{x}}$$

and we define

$$\eta_{\mathrm{eff}} \equiv \frac{\langle \sigma_{xy} \rangle}{\partial_y v_x}$$

where the averaging is over the positions in the fluid (and of the suspended particles). For simplicity, let us consider the spherical particle centered at $y = 0$.

(a) Estimate the velocity fields δv_x, δv_y and δv_z caused by the placement of the spherical particle at the origin, at a distance $r \gg a$ from the origin. Think about the symmetries of the problem to determine the dominant effects.

(b) Show that the average effect of the single suspended particle, $\langle \delta\sigma_{xy} \rangle$, can be written as

$$\langle \delta\sigma_{xy} \rangle = \frac{1}{V} \oint \mathrm{d}S_k \sigma_{kx} y$$

by using the equation $\partial_j \sigma_{ji} = 0$.

(c) Up to constant factors, estimate the above integral. Does the dominant effect come from viscous effects or from the momentum carried by the fluid?

(d) Counting the total number of suspended particles, assuming the suspension is dilute enough so that these particles do not interact, show that

$$\frac{\eta_{\mathrm{eff}}}{\eta_{\mathrm{f}}} - 1 \sim ca^3.$$

This shows that the effect of the suspension on the viscosity of the fluid as a whole is a *geometric* effect.