Viscous Drag in DNA Replication

One of the original concerns about Watson and Crick's proposal that DNA replication occurs by unzipping the double helix was that the energy required to overcome viscous effects in order to twist a very long cylinder in water would be far too large. In this problem, you will show that viscous effects are actually negligible in the untwisting of a DNA double helix.

Consider a strand of DNA, which you should approximate as a rigid cylinder of radius $R \approx 2$ nm and length $L \gg R$, suspended in water, with viscosity $\eta \approx 10^{-3}$ Pa $\cdot$ s. To unzip the helix, an enzyme called **DNA helicase** walks along one of the strands, and slowly unwinds it, twisting the entire cylinder as it goes. Approximate that it twists the cylinder at some constant angular speed ω.

We begin by computing the power rate the DNA helicase must expend in order for this process to overcome the effect of viscous drag.

(a) Use the Navier-Stokes equations to determine, approximately, the fluid velocity outside of the cylinder.

(b) From this, show that the power the DNA helicase must expend is given by

$$P = 4\pi\eta\omega^2 R^2 L.$$

Now, let's compare this answer to experimental data. Like pretty much everything in biology, DNA helicase is powered by ATP. Every time an ATP molecule is dephosphorylated, it releases about $\Delta G_{\text{ATP}} \approx 8 \times 10^{-20}$ J. Suppose that n ATP molecules are needed to power the DNA helicase to twist the double helix by one revolution (2π radians).

(c) What is the length L at which the DNA helicase must expend more than 1 ATP per revolution in order to overcome viscous drag?

(d) A typical bacterial DNA has about 10^6 to 10^7 base pairs (meaning A, C, G or T in the nucleic acid sequence). If each base pair corresponds to about 0.3 nm, comment on whether or not the bacterial DNA is short enough so that viscous effects are not important.