Fluid Dynamics of an Explosion

In this problem we will consider a simple model for the fluid dynamics of a gas during an explosion. Let us assume that the explosion is spherically symmetric, which will simplify things significantly (although in reality this is not usually such a good assumption).

There is no naïve length scale for this problem, so let us make the ansatz that this is a similarity flow, with all functions only of the parameter:

$$z = \frac{r}{t}.$$

(a) Assuming that $\partial_i P = c^2 \partial_i \rho$, spherical symmetry, and the similarity flow, show that the equations of mass and momentum conservation boil down to the single first order differential equation:

$$\left(\frac{(z-v)^2}{c^2} - 1\right)\frac{\mathrm{d}v}{\mathrm{d}z} = \frac{2v}{z}.$$

(b) Why should $v = 0$ for $z > c$? Give a simple physical justification.

(c) This does not admit an analytical solution. However, one can show that if the speed of sound in the gas is c_0, then we can take $v + c \approx c_0 + \alpha_0 v$. If we make this approximation, and only keep the first order terms in v and $c - z$, show that, up to an unknown constant of integration A, the solution to the above differential equation is, for $z < c$

$$z - c(v) - v = \alpha_0 v \log \frac{A}{v}.$$

(d) It is reasonable to physically assume that $v \geq 0$. Argue that, combining the solutions of the previous two parts, v is only non-zero for a *finite* region of z. Describe what this region of z is, to the fullest extent you can by hand, and then sketch $v(z)$ carefully. Comment on the result, being sure to focus on what is happening at any regions of transition in the dynamics.