Height of a Sequoia

The sequoias of California are the tallest trees in the world; they can reach 100 m in height. This is actually a remarkable feat of nature. To understand why, we need to think about the transport of water up the tree. To survive, water must reach the top of the tree. In a static situation, the **water potential**, defined as

$$\psi = P + \rho g z - nT$$

must be a constant function of z, where P is hydrostatic pressure, z is the distance up the tree, $T \approx 4 \times 10^{-21}$ J is the temperature of the water, and n is a concentration of solutes (such as sugars) in the water.

(a) Neglect the chemical effects of the water potential (set $n = 0$). Suppose that $P = 0$ at the top of the tree. How high could a sequoia be? Use the fact that the pressure at $z = 0$ should be atmospheric pressure: $P_0 \approx 10^5$ Pa.

(b) Obviously, sequoias must come up with a more clever mechanism. Now, let us see what would happen if the tree places dissolved ions in the water, to lift up water to the top of the plant. How many ions must be dissolved per volume at the base of the tree if this is the mechanism for lifting water? Compare to the number of water molecules: $\approx 5 \times 10^{28}$ m^{-3}.

(c) The true mechanism for lifting water up the sequoia is **transpiration**, due to water evaporating at the surface of "leaves". The evaporation pressure of water is about 2 kPa at temperature T. What must be the ratio of the (effective) surface area of the leaves, to the cross sectional area of the trunk (at least, the parts carrying water), in order for transpiration to bring water to the top of the tree? You can be heuristic. Comment on if the answer is reasonable.