Banks-Zaks Fixed Point

Consider a non-Abelian gauge theory with gauge group G and fermions in the representation R. We can write the 1 loop β function for such a theory is given by

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2b_0$$

where $t = \log\mu$, $x = 16\pi^2/g^2$, and

$$b_0 = \frac{4}{3}\mathrm{T}_2(R) - \frac{11}{3}\mathrm{C}_2(G).$$

If we include 2 loop corrections, we would find that

$$\frac{\mathrm{d}x}{\mathrm{d}t} = -2b_0 - \frac{2b_1}{x},$$

with

$$b_1 = \left(\frac{20}{3}\mathrm{C}_2(G) + 4\mathrm{C}_2(R)\right)\mathrm{T}_2(R) - \frac{34}{3}\mathrm{C}_2(G)^2.$$

A fixed point x^* of a theory occurs when $\mathrm{d}x(x^*)/\mathrm{d}t = 0$. Consider the general form for $\mathrm{d}x/\mathrm{d}t$ in terms of b_0 and b_1. In order for a fixed point to exist in this theory, we must have $b_0/b_1 < 0$, so assume that this is the case.

(a) Find an expression for x^*.

(b) If $b_0 < 0$, sketch $x(t)$ for $x(t_0) > x^*$ and $x(t_0) < x^*$.

(c) If $b_0 > 0$, sketch $x(t)$ for $x(t_0) > x^*$ and $x(t_0) < x^*$.

(d) Depending on the sign of b_0, the fixed point will either be an IR or UV fixed point. Which one is which?

Consider a $\mathrm{SU}(N)$ gauge theory with F fermions in the fundamental representation.

(e) Find b_0 and b_1.

(f) For what F does a fixed point exist?

(g) Will the fixed point be a IR or UV fixed point? The fixed point is called the **Banks-Zaks fixed point**.

(h) If g^* is the value of g at the fixed point, find the smallest possible value of g^* for fixed N and variable F.