quantum field theory \rightarrow gauge theory

Stueckelberg Mechanism

We showed earlier that adding a mass term for the vector field in a gauge theory spoils gauge invariance. However, there is a way to include an extra field into the action such that the resulting action is gauge invariant and includes a massive vector field, called the Stueckelberg action. Consider the Lagrangian

$$\mathcal{L} = -\frac{1}{2}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}m^2\left(\partial_\mu\phi + A_\mu\right)\left(\partial^\mu\phi + A^\mu\right)$$

with ϕ a real scalar field.

- (a) Show that if the field ϕ transforms under a gauge transformation as well as A, then the action can be made gauge invariant.
- (b) Show that the Stueckelberg mechanism does not work for giving mass to a non-Abelian gauge theory.