Polaritons

In this problem we explore how the interaction of phonons with an electromagnetic field creates fundamental excitations called **polaritons** in a solid. As in class, you can assume that the phonons are on a 1D lattice with atoms of mass m and lattice spacing a. In addition, assume the ions have charge $+e$, and denote by v the volume of the unit cell (we can take $v = a^3$, if you like).

(a) Begin by including the effects of an electromagnetic field interacting with the atoms in the lattice. By letting A_j be the relevant component of the electromagnetic vector potential at lattice site j (neglecting the relative motion of the lattice sites), show that

$$L = \sum_j \left[\frac{1}{2}m\dot{x}_j^2 - \frac{1}{2}m{\omega_0}^2\left(x_{j+1} - x_j\right)^2 + e\dot{x}_j A_j + \frac{\epsilon_0 v}{2}\dot{A}_j^2 - \frac{\epsilon_0 v c^2}{8a^2}\left(A_{j+1} - A_j\right)^2\right].$$

(b) Find the equations of motion.

(c) Use translational symmetry to find the normal modes, and show that the dispersion relations for these 2 modes are given by

$$\omega^2 = \frac{{\omega_\mathrm{p}}^2 + (c^2+u^2)k^2 \pm \sqrt{(c^2-u^2)^2k^4 + 2(c^2+u^2)k^2{\omega_\mathrm{p}}^2 + {\omega_\mathrm{p}}^4}}{2}$$

and find expressions for ω_p (called the plasma frequency) and u (the phonon speed).