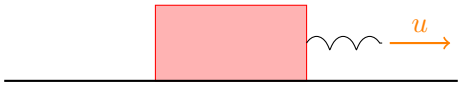
classical mechanics → harmonic oscillation ★★★

Stick-Slip Motion

Consider the following scenario: a block of mass m, and weight mg, sits on a floor. The coefficient of static friction between the block and the floor is μ_s, and the coefficient of kinetic friction is μ_k: as usual, assume that $\mu_s > \mu_k$. The block is attached to a spring, with spring constant $m\omega^2$. At time $t = 0$, the block is at rest, and the free end of the spring is pulled to the right at constant velocity u.[1]



Let $z(t)$ be the displacement of the block from its initial position; let $x(t)$ be the (stretched) length of the spring. Set $z(0) = x(0) = 0$.

(a) Using kinematics and Newton's laws, determine relations between $\dot{z}$ and $\dot{x}$, and find equations for $\ddot{z}$ and $\ddot{x}$.

(b) For large times, show that the block will undergo periodic motion. Provide *accurate* sketches of both $x(t)$ and $z(t)$.

(c) Finding a formula for the period of the oscillations is nasty, so you don't need to do it. However, do determine whether or not the period of stick-slip oscillations is shorter or longer than the period in the case of motion with only slip.

Experimentally, we find that when a stick-slip system is pulled with a high enough velocity u_c, the stick-slip motion stops and becomes only "slip" motion. A simple explanation for this is that, for a given oscillation of amplitude A in the slip phase, dissipative forces will decrease the amplitude after one period to $(1 - \alpha)A$, for $\alpha \ll 1$.

(d) Determine the value of u_c in terms of $\omega, g, \mu_s, \mu_k, \alpha$.

(e) If $u < u_c$, will the stick-slip oscillations be damped? What about if $u > u_c$?

Stick-slip motion is an important phenomena in many mechanical applications, from the stick-slip motion of faults which leads to earthquakes, to the squeaking motion of chalk, to the creaking of doors.

[1] This problem is adapted from a physics olympiad problem.