Chimneys

A chimney is a simple device used to allow hot gas to escape from a fire, or other sort of object which heats up gas. It turns out that chimneys must be built to a certain height or they will not function properly: in this problem, you will show why with a simple model.

Consider a chimney of height h and cross-sectional area A, placed over a fire. The hot gas rising from the fire, which is "created" at volume rate r per unit time, is at temperature T_f, and the air temperature outside the chimney is T_a. Assume that the hot gas in the chimney has the same molecular mass as the air outside the chimney, and assume that the pressure of the hot gas at the fire is the same as atmospheric pressure at that height. Assume that the density and temperature of the gas in the chimney is roughly independent of height, as is the density and temperature of the air outside.

(a) Using the basic equations of fluid dynamics and assuming that the gases are ideal, show that

$$h \geq \frac{r^2}{2gA^2}\frac{T_\mathrm{a}}{T_\mathrm{f}-T_\mathrm{a}}$$

in order for the chimney to function properly – i.e., for the hot gas to escape from the top.

(b) Suppose an industrial furnace operates at 1000 K, and emits 3 m^3/s worth of hot gas into the chimney, which has a diameter of 1 m. Using $g \approx 10$ m/s and $T_\mathrm{a} \approx 300$ K, determine the minimum height of h. Is it reasonable?