Spectral Line Broadening

We know from quantum mechanics that a quantum gas will typically emit radiation at precise wavelengths. However, if the atom is moving relative to the observer, there will be a Doppler shift to the photon. Indeed, one way to measure the temperature of a gas is to measure the "thickness" of a well-defined emission line, as you will show in this problem.

(a) Suppose that the gas is at temperature T, and the emissions are coming from an atom with rest energy mc^2. Show that, if the emission line has wavelength λ_0 in the rest frame of the atom, we expect an intensity distribution of the form

$$I(\lambda) \sim \exp\left[-\frac{mc^2}{2k_{\mathrm{B}}T}\left(\frac{\lambda-\lambda_0}{\lambda_0}\right)^2\right].$$

(b) Suppose we wish to determine the temperature of a planetary nebula with this method. A typical planetary nebula has a temperature of $T \approx 10^4$ K. Using that the mass of a hydrogen atom is approximately 1.7×10^{-27} kg, find the expected value of λ_{w}, the standard deviation of the Gaussian distribution above.