Magnetic Helix

In some ferromagnets, the Landau free energy at long length scales can be approximated by:

$$F = \int \mathrm{d}^3x \left[\frac{\rho}{2}\partial_j M_i \partial_j M_i + \frac{a}{2}M_i M_i + \frac{\alpha}{2}\epsilon_{ijk}M_i\partial_j M_k \right]$$

where $M_i(x)$ is the 3 component magnetization vector of the ferromagnet.

To find the minimum of this free energy, make the ansatz that

$$M_i(x) = A_i \cos(k \cdot x) + B_i \sin(k \cdot x).$$

(a) Using the ansatz above, find an expression for F.

(b) What is the relative orientation of the vectors A, B, and k, in the free energy minimum?

(c) Show that there is an interesting phase transition as a is tuned below a critical value a_c. What happens? Find the value of a_c and sketch the magnetization in the interesting phase.