

Mixtures of Superfluid Helium

In the spirit of Landau theory, we can write down a free energy density for a mixture of ^3He and ^4He superfluid:

$$\mathcal{F} = \frac{t}{2}|\psi|^2 + u|\psi|^4 + v|\psi|^6 + \frac{c^2}{2\sigma^2} - \gamma c|\psi|^2.$$

Here, $\psi \in \mathbb{C}$ is an order parameter corresponding to the ^4He , which corresponds to a macroscopic quantum state of the superfluid. $c \in \mathbb{R}^+$ is an order parameter corresponding to the concentration of ^3He impurities in the superfluid. u, v, γ and σ are positive constants.

- (a) Remove c (i.e., determine c in terms of $|\psi|^2$), thus expressing \mathcal{F} in terms of ψ only, by using the mean field equations.
- (b) Show that there are two phases: an ordered phase of $\psi \neq 0$, and a disordered phase of $\psi = 0$. Draw the phase diagram between the two phases, carefully describing the boundary between the two phases. Be sure to note where the phase transition is continuous or discontinuous.