

Extrema of the Action

In this problem, we will ask the question: is the action of the classical path which we find by the principle of least action *actually* a minimum? For simplicity, we will consider

$$L = \frac{\dot{x}^2 - \omega^2 x^2}{2},$$

the Lagrangian of the harmonic oscillator.

- (a) Begin by using the fact that all solutions of the principle of least action for this Lagrangian are of the form

$$x(t) = A \cos(\omega t + \phi).$$

Assume we are looking for paths with $t \in [0, T]$. Find the action S_0 of the path described by this solution.

- (b) Now, determine whether or not the action found above is a minimum, a maximum, or a saddle. Note that the answer may depend on the choices of ω and T .
- (c) Can the action ever be a maximum on the classical path? Here you may use a Lagrangian other than the one above.