Tracing Magnetic Field Lines

Consider a magnetic field given by

$$\mathbf{B} = B_0\hat{\mathbf{z}} + \nabla \times \mathbf{A}$$

where $\mathbf{A} = A_0(x, y, z)\hat{\mathbf{z}}$.

(a) Find the components B_x, B_y, and B_z of $\mathbf{B}$.

(b) Now, let $\mathbf{r}(z)$ trace a magnetic field line – i.e., it is the curve such that the tangent line $\partial_z\mathbf{r}$ is parallel to $\mathbf{B}(\mathbf{r})$. Show that the curves $x(z)$ and $y(z)$ can be found by solving Hamilton's equations with $H = A_0/B_0$. Of x and y, which one is the "position" and which is the "momentum"?

Typically, we find applications of Lagrangian mechanics outside of physics, since variational calculus is a very universally useful skill. Applications such as this where Hamilton's equations come up outside of physics are much rarer.