Random Walk on a Group

One way to define naturally a random walk on a group, G, is by a Markov chain with state space G, and transition probabilities

$$P(g, hg) = \mu(h)$$

for $h \in G$, and some fixed function μ. In this problem, assume that G is a finite group, for simplicity.

Some of the examples of random walks we have seen can be cast in this form. For example, the random walk on $\mathbb{Z}_n$ can be easily recast as the random walk on the group $\mathbb{Z}_n$, with $\mu(-1) = \mu(1) = 1/2$.

As you will now show, if we can write a Markov chain as a random walk on a finite group, it is very easy to extract some nice properties.

(a) Show that the uniform distribution on G is stationary.

(b) Show that the random walk is irreducible $\iff \langle h | \mu(h) > 0 \rangle = G$.

(c) Define μ to be symmetric if $\mu(g) = \mu(g^{-1})$. Show that μ is symmetric $\iff$ P is reversible. You can assume that the random walk is irreducible.