# Asteroid Impact

In this problem, we will get a heuristic understanding of the energies involved in asteroid impacts on Earth, and the relative danger of asteroid impacts of various sizes.

Consider an asteroid in deep space to be a sphere of radius $R_0$ and mass density $\rho_\mathrm{a} \approx 6000$ kg/m$^3$. Estimate that the speed of this asteroid is quite small before it gets near the Earth. When it gets near the Earth, it begins to become attracted by Earth's gravitational field, and will fall towards the Earth. You may use that $G \approx 7 \times 10^{-11}$ J $\cdot$ m/kg$^2$, $M_\mathrm{E} \approx 6 \times 10^{24}$ kg, and $R_\mathrm{E} \approx 7 \times 10^6$ m. The height of the Earth's atmosphere is approximately $H \approx 10^5$ m.

(a) Argue that the Earth's gravitational pull is a negligible effect once the asteroid enter's the Earth's atmosphere.

(b) How fast is an asteroid going when it enters the Earth's atmosphere? Find an expression in terms of $\rho_\mathrm{a}$, $G$, $M_\mathrm{E}$, $H$ and $R_\mathrm{E}$, and evaluate it in terms of the parameters above.

Once the asteroid enters the atmosphere, it begins to experience a drag force due to the atmosphere. If $\rho_\mathrm{g} \approx 1$ kg/m$^3$ is the density of the atmosphere,[1] and $A$ is the effective surface area of the asteroid, we can estimate the drag force on the asteroid, traveling at velocity $v$, by

$$F \approx -\rho_\mathrm{a} A v^2.$$

(c) Find the velocity of the asteroid on impact, and calculate the energy of the asteroid on impact.

(d) Estimate the radius of an asteroid for which the atmosphere will significantly slow it on impact. Scientists have argued that asteroids with size $R_0 > 50$ m can cause significant damage on impact; compare your answer to this number.

(e) For what size asteroid will the energy on impact be equivalent to a nuclear (fusion) weapon, which releases $5 \times 10^{15}$ J on impact?

Small asteroids vaporize when they enter the Earth's atmosphere. This takes a *lot* of energy: we will for simplicity assume that the asteroids directly vaporize, and that the energy to vaporize the asteroid is $c = 10^7$ J/kg. Obviously the entire asteroid does not vaporize at once, but it slowly burns up as it goes, with the outside of the asteroid vaporizing as it falls.

(f) Find an equation of motion for $\mathrm{d}R/\mathrm{d}t$, assuming that a fraction $\beta$ of the energy lost as the asteroid falls through the atmosphere heats up the asteroid, and a fraction $1-\beta$ heats up the nearby gas. Give a heuristic reason why the equation for $\mathrm{d}v/\mathrm{d}t$ should remain unchanged from part (c).

(g) Combine the equations above to show that

$$v_0^2 - v^2 = \frac{6c}{\beta} \log \frac{R_0}{R}.$$

Then reduce the dynamics to a single first order equation for $v$, and comment on the resulting dynamics (you do not need to find an exact solution).

(h) Argue that the conclusion from part (e) is essentially unchanged, by plugging in for $c$ and $v_0$. Heuristically comment on what happens to asteroids with small $R_0$ when vaporization is taken into account.

[1] In reality, there is a very strong dependence on height of the density – you may neglect this effect.